Introduction To Small Signal And Averaged Switch Modeling

Robert V. White
bob.white@embeddedpowerlabs.com

Seminar Prepared For APEC 2015

Seminar Introduction

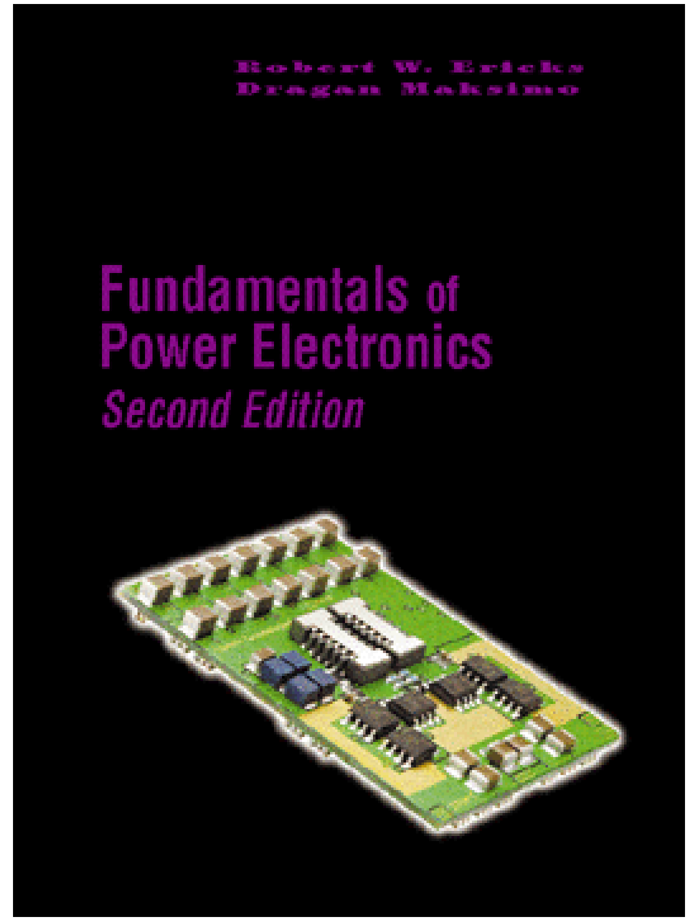
- This Is An Introductory Seminar
  - Why Do We Need Small Signal Modeling?
  - Small Signal Model Derivation
  - Deriving Transfer Functions From The Small Signal Model
  - Averaged Switch Modeling
- Detailed Examples (All The Algebra)

Seminar Introduction: Not Discussed

- Models For Discontinuous Conduction Mode
- Control Loop Design
  - Error Amp/Compensator Design
  - Pole-Zero Placement
  - Loop Stability

Seminar Introduction

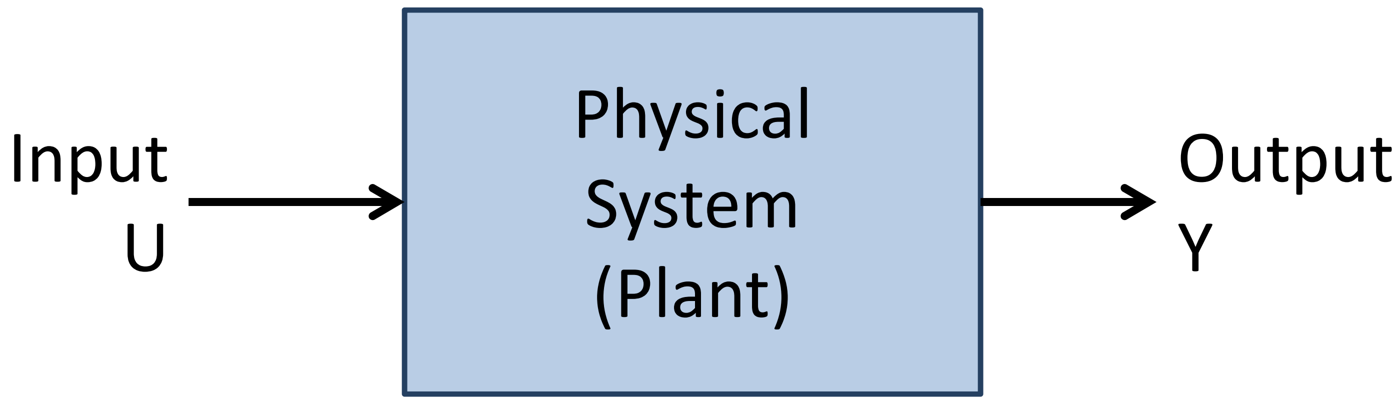
- Much Of This Seminar Is Based On Material From "Fundamentals Of Power Electronics", 2nd Edition, Erickson & Maksimovic, Kluwer Academic Publishers, ISBN 0-7923-7270-0
  - Chapter 7, AC Equivalent Circuit Modeling
  - Chapter 8, Converter Transfer Functions



Seminar Introduction

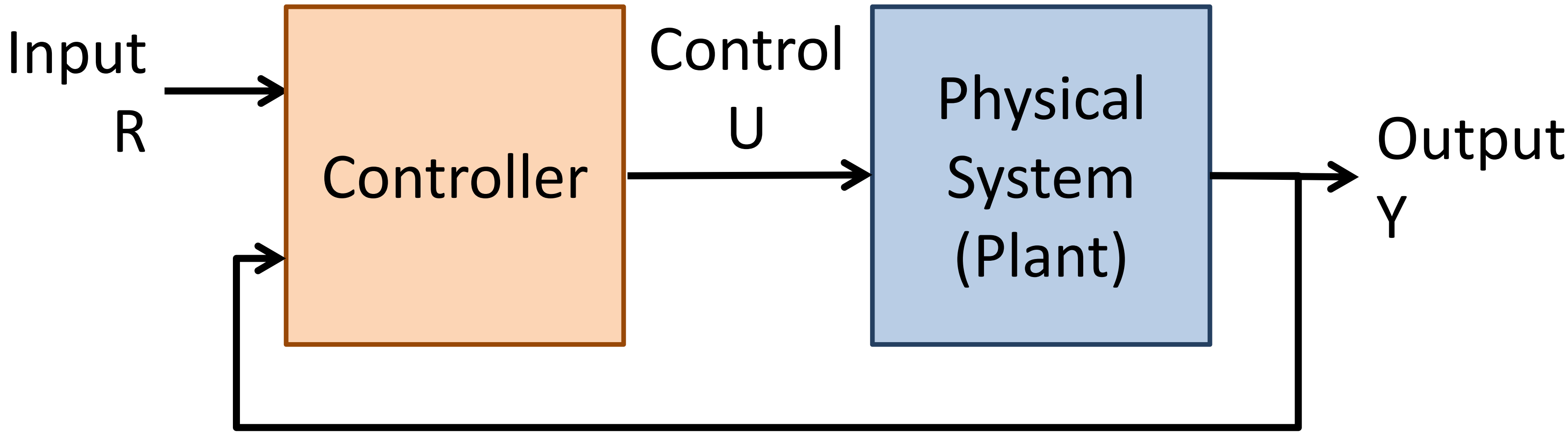
- Timing
  - 90 Minutes Presentation
  - 30 Minute Break
  - 90 Minutes Presentation
- Ask Questions At Any Time
- Fill Out The Survey Form!

Controls 101 Review



Issue: The Actual Output
Is Not The Desired Output

Feedback



Goals:

- Make the output track the reference input
- Reject disturbances

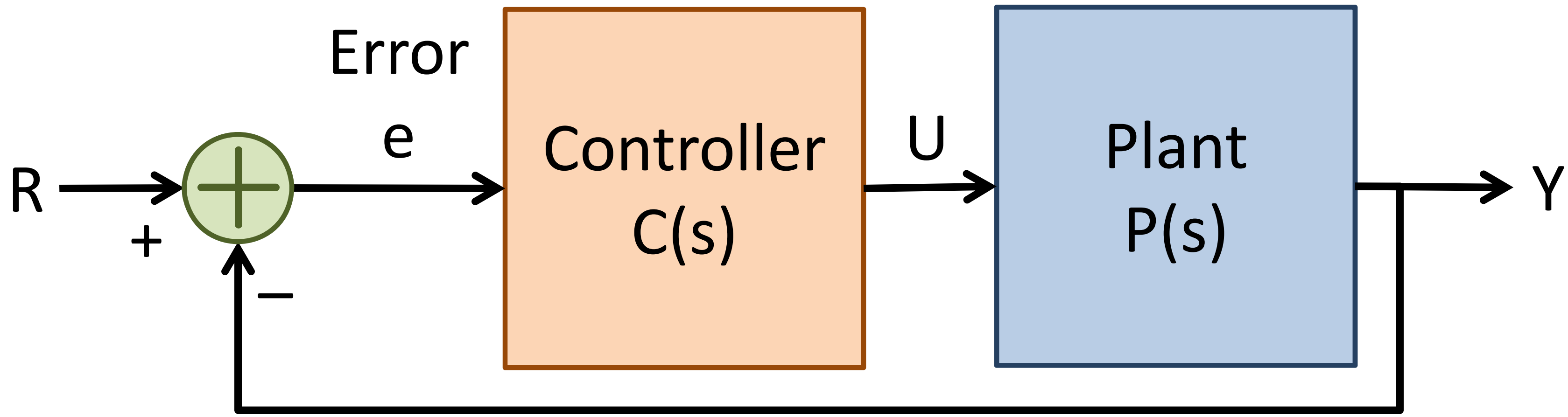
Requirement: Remain stable at all times

Stability

- Bounded Input-Bounded Output (BIBO)
  - Finite input results in finite output

- Lyapunov
  - System has an equilibrium point or points
  - Small disturbance from equilibrium results in small change in output
  - Exponential: Output change decays in time

Design And Analysis



- Time domain solutions too tedious
- Transform to frequency or s domain

$$Y(s) = \frac{P(s) \cdot C(s)}{1 + P(s) \cdot C(s)} \cdot R(s) \qquad H(s) = \frac{Y(s)}{R(s)} = \frac{P(s) \cdot C(s)}{1 + P(s) \cdot C(s)}$$

Design And Analysis

R —— → Y

Stable Means

This Term Can Never Be Equal To Zero

H(s) Has No Poles In RHP

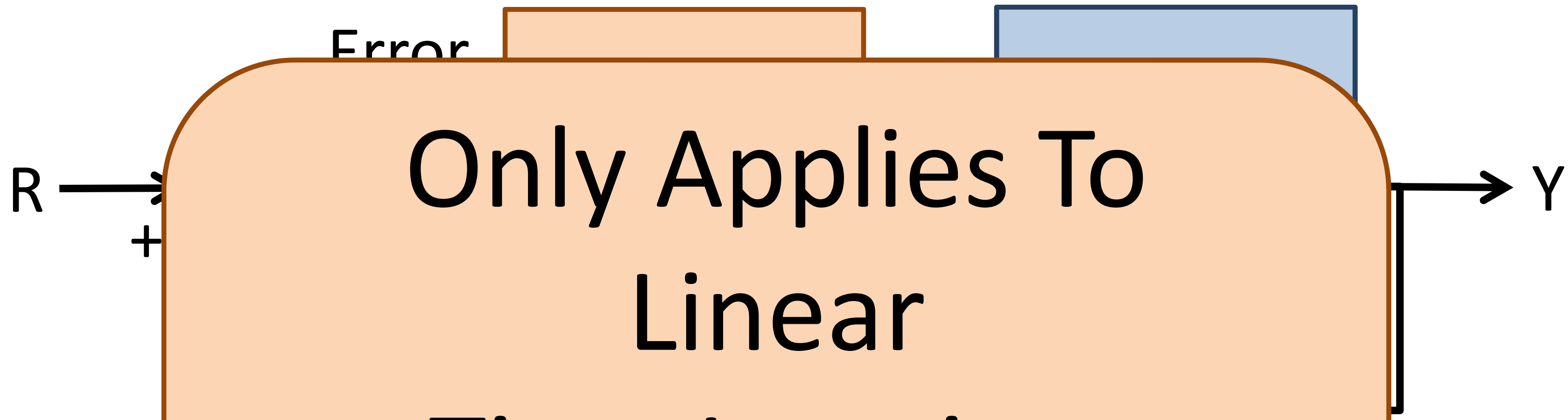
No RHP Poles In P(s) Or C(s)

Cancelled By Matching RHP Zeroes

$$Y(s) = \frac{P(s) \cdot C(s)}{1 + P(s) \cdot C(s)} \cdot R(s)$$

$$H(s) = \frac{Y(s)}{R(s)} = \frac{P(s) \cdot C(s)}{1 + P(s) \cdot C(s)}$$

Design And Analysis

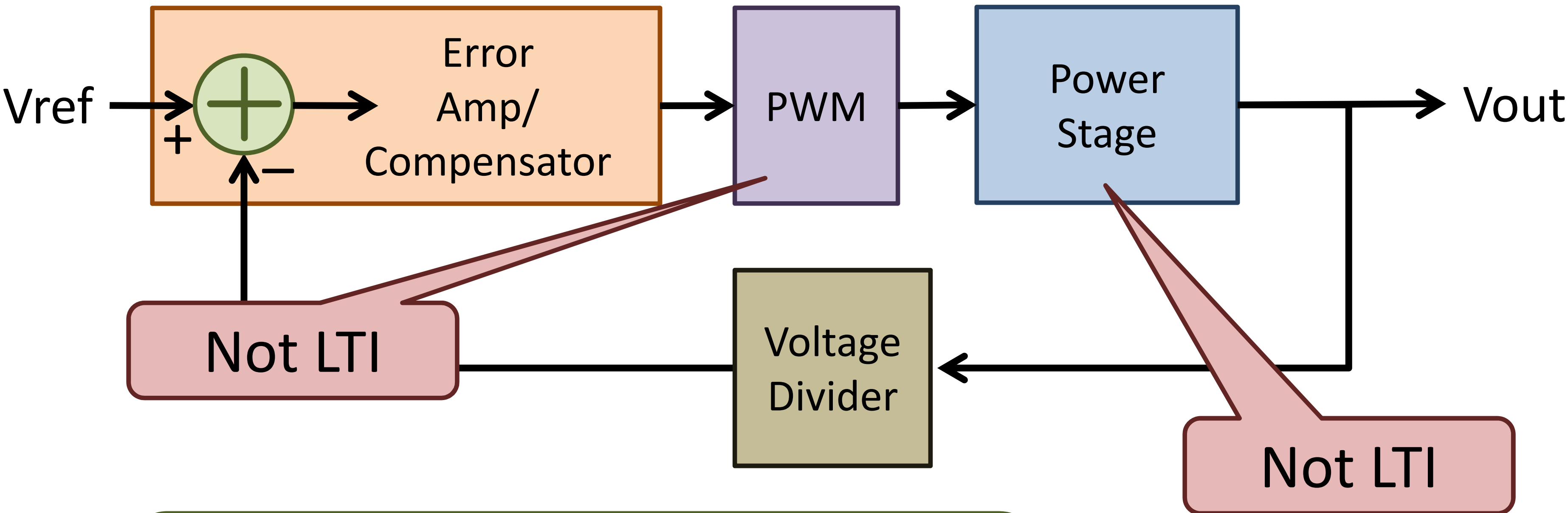


Only Applies To
Linear
Time Invariant
(LTI)
Systems!

$Y(s) = \frac{\ldots}{1+P(s)\cdot C(s)}$ … $\frac{\ldots)\cdot C(s)}{R(s) \quad 1+P(s)\cdot C(s)}$

EMBEDDED POWER LABS

Power Supply Model



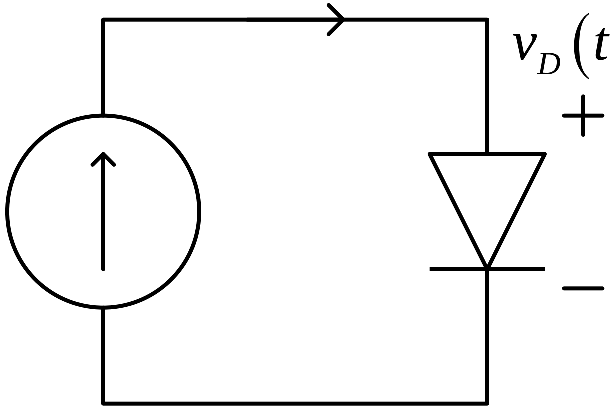
We Need Circuit Models
Valid In The s-Domain

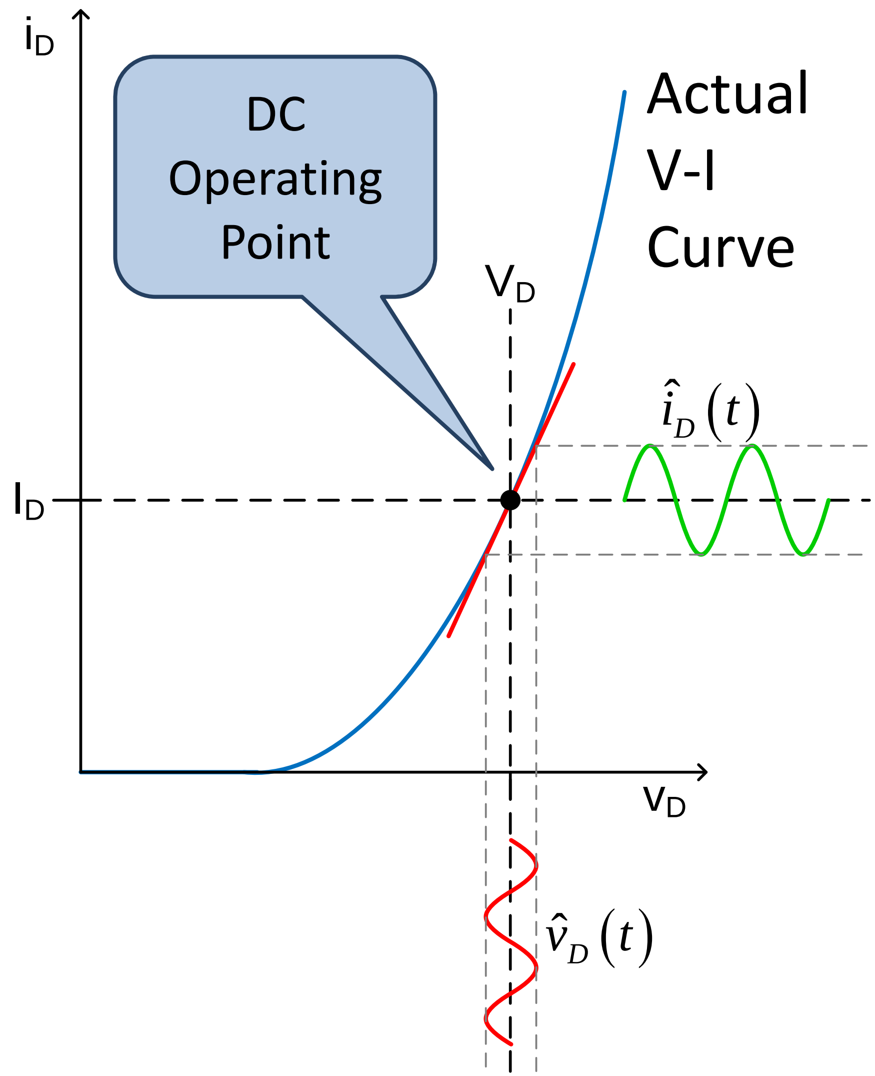
EMBEDDED POWER LABS

Small Signal Modeling

$$i_D(t) = I_D + \hat{i}_D(t)$$

$$v_D(t) = V_D + \hat{v}_D(t)$$





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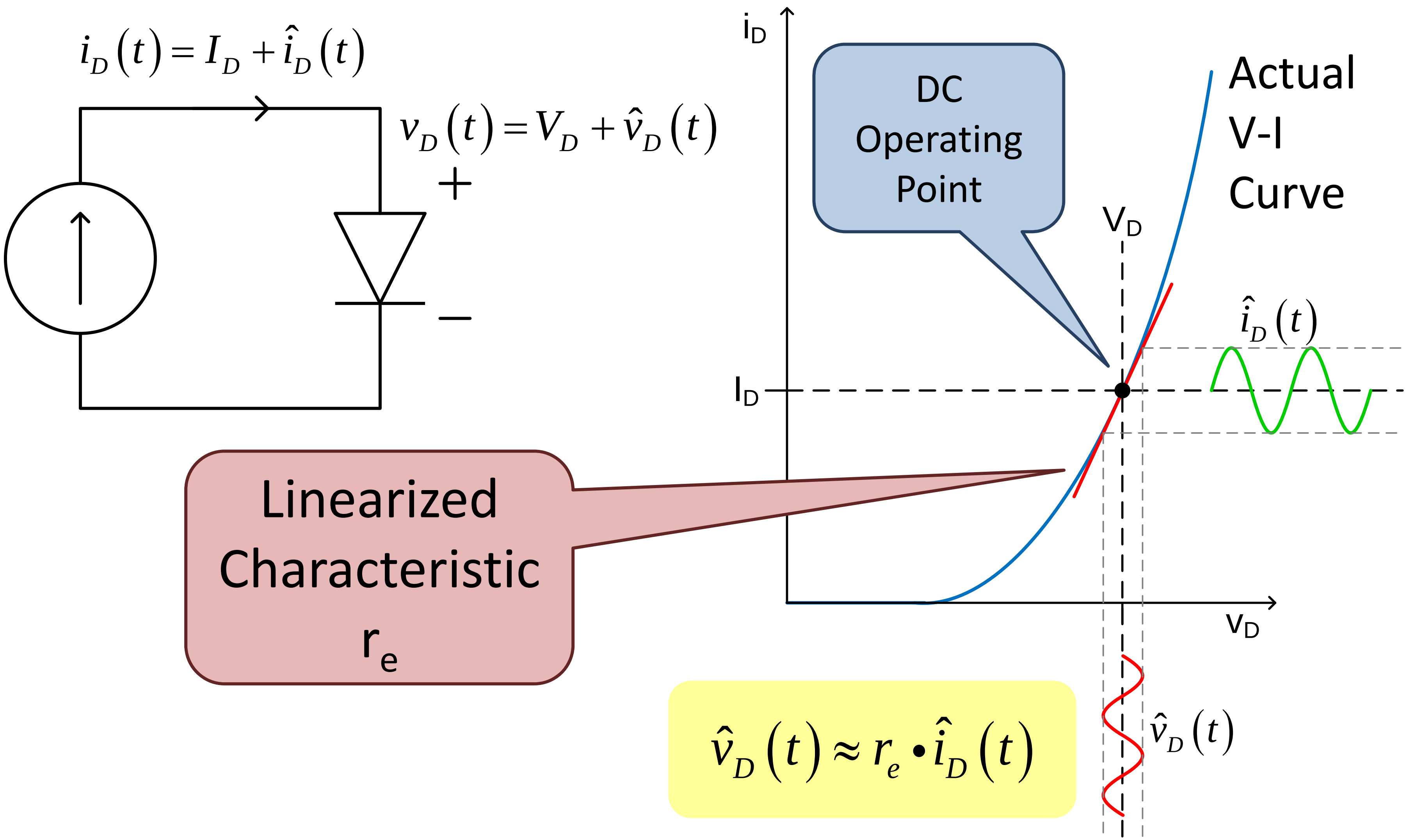
Small Signal Modeling



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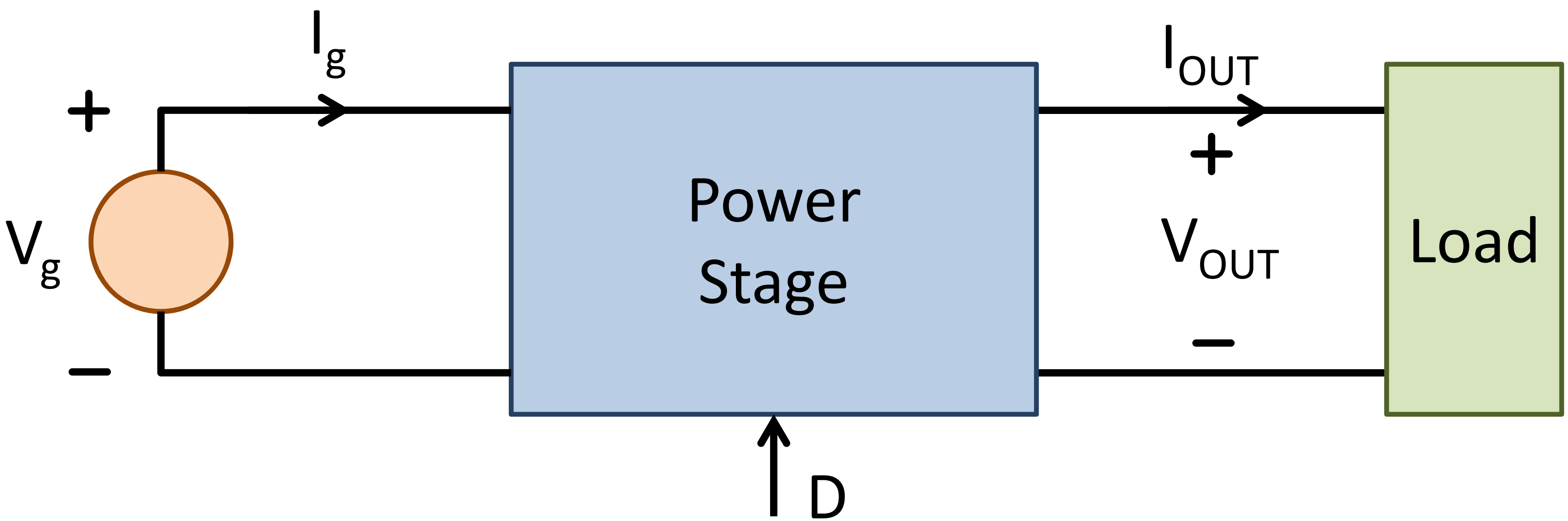
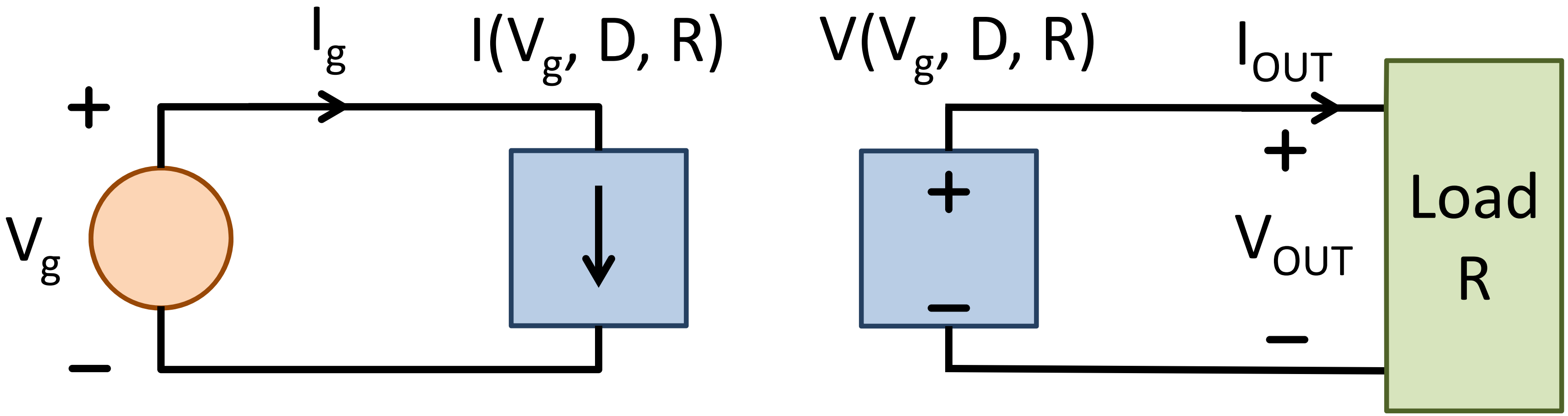
DC Model



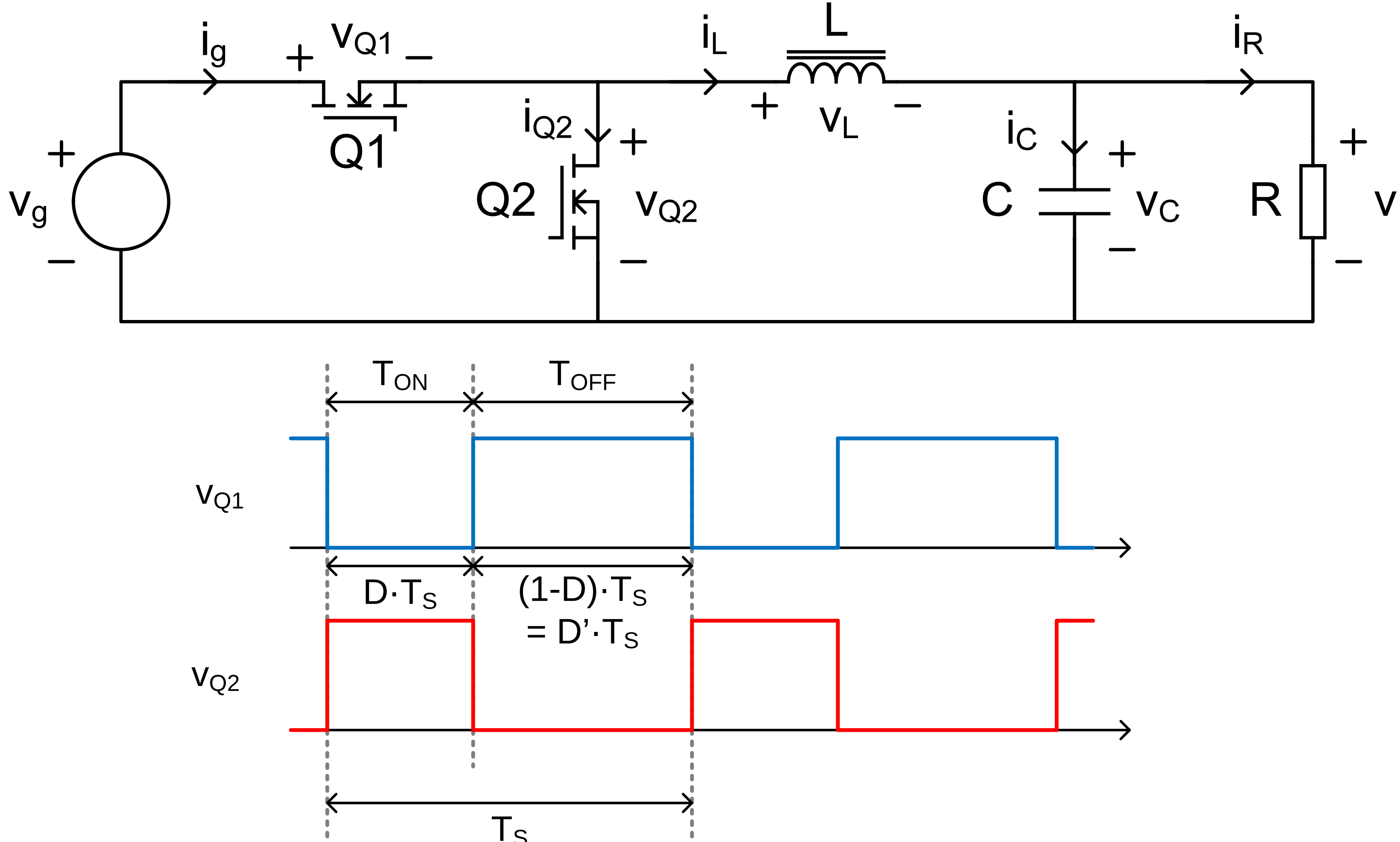
Inputs
- Input Voltage, V_g
- Control Input, D
- Load
  - Resistance, R
  - Current, I_{LOAD}
  - Voltage, V_{LOAD}

Outputs
- Output Voltage, V_{OUT}
- Input Current, I_g

DC Model



Buck Converter DC Model



Solving The Output Voltage

$$\langle v_L(t)\rangle_{Ts} = \langle v_L(t)\rangle = \frac{1}{T_S}\cdot\int_0^{T_S} v_L(t)\,dt = 0$$

Inductor Volt-Second Balance

$$\frac{1}{T_S}\cdot\int_0^{T_S} v_L(t)\,dt = \frac{1}{T_S}\left(\int_0^{Ton} v_L(t)\,dt + \int_{T_{ON}}^{T_S} v_L(t)\,dt\right) = 0$$

$$\int_0^{Ton}\left(v_g - v_C(t)\right)dt + \int_{T_{ON}}^{T_S}\left(-v_C(t)\right)dt = 0$$

Small Ripple Approximation

$$v_g(t) = V_g + \tilde{v}_g(t)$$

$$\left|\tilde{v}_g(t)\right| \ll V_g$$

$$v_g(t) = V_g + \tilde{v}_g(t) \approx V_g$$

$$v_C(t) = V_C + \tilde{v}_C(t)$$

$$\left|\tilde{v}_C(t)\right| \ll V_C$$

$$v_C(t) = V_C + \tilde{v}_C(t) \approx V_C$$

Solving The Output Voltage

$$\int_{0}^{Ton}\left(V_g - V_C\right)dt + \int_{T_{ON}}^{T_S}\left(-V_C\right)dt = 0$$

$$\left(V_g - V_C\right)\cdot T_{ON} - V_C\cdot\left(T_S - T_{ON}\right) = 0$$

$$\left(V_g - V_C\right)\cdot D\cdot T_S - V_C\cdot\left(T_S - D\cdot T_S\right) = 0$$

$$\left(V_g - V_C\right)\cdot D - V_C\cdot\left(1 - D\right) = 0$$

$$D\cdot V_g - DvV_C - V_C + D\cdot V_C = 0$$

$$D\cdot V_g - V_C = 0$$

$$V_C = D\cdot V_g$$

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Solving The Inductor Current

$$\left\langle i_C(t)\right\rangle_{Ts} = \left\langle i_C(t)\right\rangle = 0$$

Capacitor Charge Balance

$$\frac{1}{T_S}\cdot\int_0^{T_S} i_C(t)\,dt = \frac{1}{T_S}\cdot\left(\int_0^{Ton} i_C(t)\,dt + \int_{T_{ON}}^{T_S} i_C(t)\,dt\right) = 0$$

$$\int_0^{Ton} i_C(t)\,dt + \int_{T_{ON}}^{T_S} i_C(t)\,dt = 0$$

$$\int_0^{Ton}\left(i_L(t) - \frac{1}{R}\cdot v_C(t)\right)dt + \int_{T_{ON}}^{T_S}\left(i_L(t) - \frac{1}{R}\cdot v_C(t)\right)dt = 0$$

$$\int_0^{T_S}\left(i_L(t) - \frac{1}{R}\cdot v_C(t)\right)dt = 0$$

Solving The Inductor Current

$$v_C(t) \approx V_C$$

$$i_L(t) = I_L + \tilde{i}_L(t)$$

$$\left|\tilde{i}_L(t)\right| \ll I_L$$

$$i_L(t) = I_L + \tilde{i}_L(t) \approx I_L$$

Small Ripple Approximation

$$\int_0^{T_S} \left(I_L - \frac{1}{R} \cdot V_C \right) dt = 0$$

$$\left(I_L - \frac{1}{R} \cdot V_C \right) \cdot T_S = 0$$

$$I_L - \frac{1}{R} \cdot V_C = 0$$

$$I_L = \frac{1}{R} \cdot V_C$$

EMBEDDED POWER LABS

Solving The Input Current

$$\left\langle i_g(t) \right\rangle_{Ts} = \left\langle i_g(t) \right\rangle = I_g = \frac{1}{T_S} \cdot \int_0^{T_S} i_g(t)\,dt$$

$$I_g = \frac{1}{T_S} \cdot \left(\int_0^{T_{ON}} i_g(t)\,dt + \int_0^{T_{OFF}} i_g(t)\,dt \right)$$

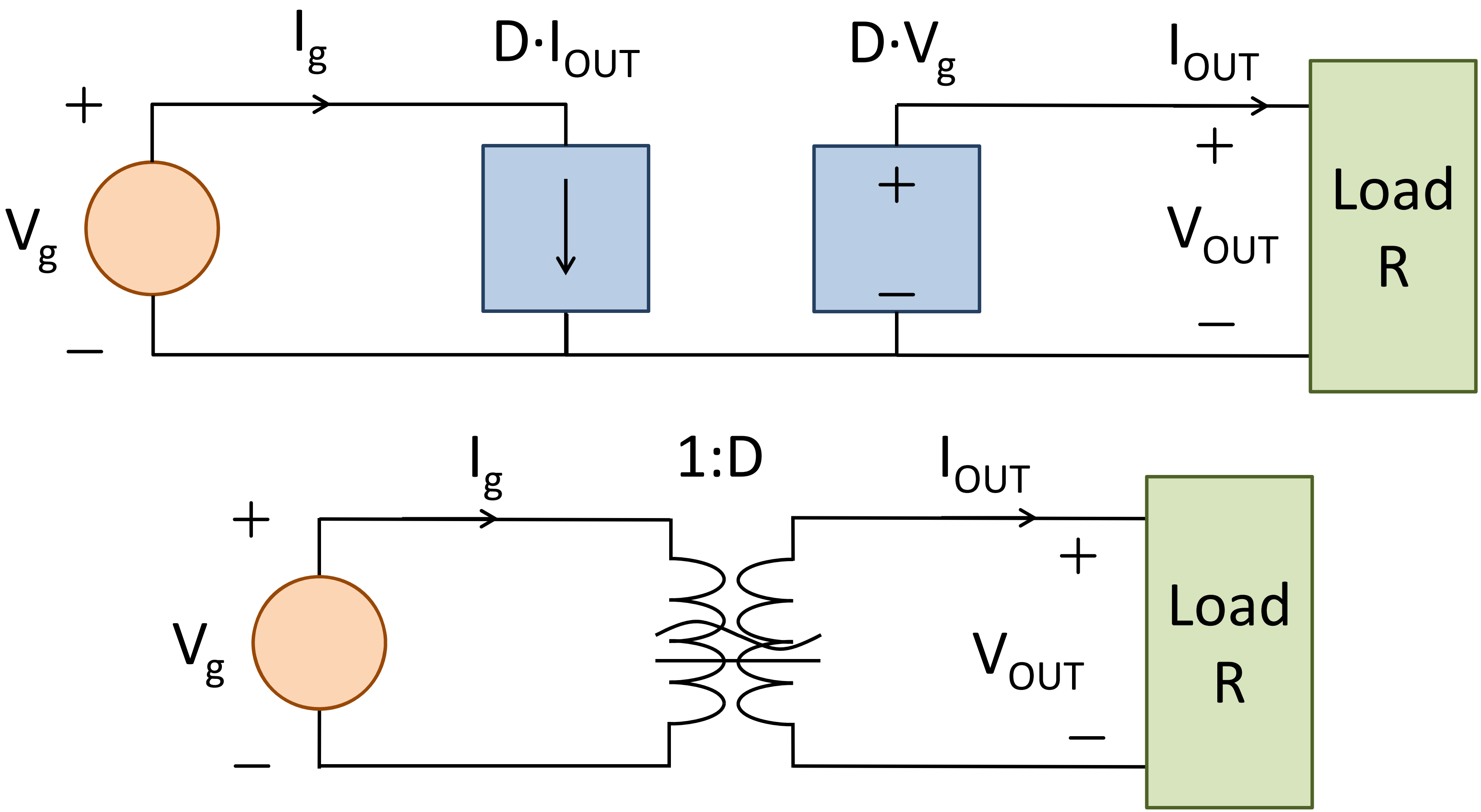
$$= \frac{1}{T_S} \cdot \left(\int_0^{T_{ON}} i_g(t)\,dt + \int_0^{T_{OFF}} 0\,dt \right) = \frac{1}{T_S} \cdot \int_0^{T_{ON}} i_g(t)\,dt$$

$$i_g(t) = i_L(t) \approx I_L$$

From The Small Ripple Approximation

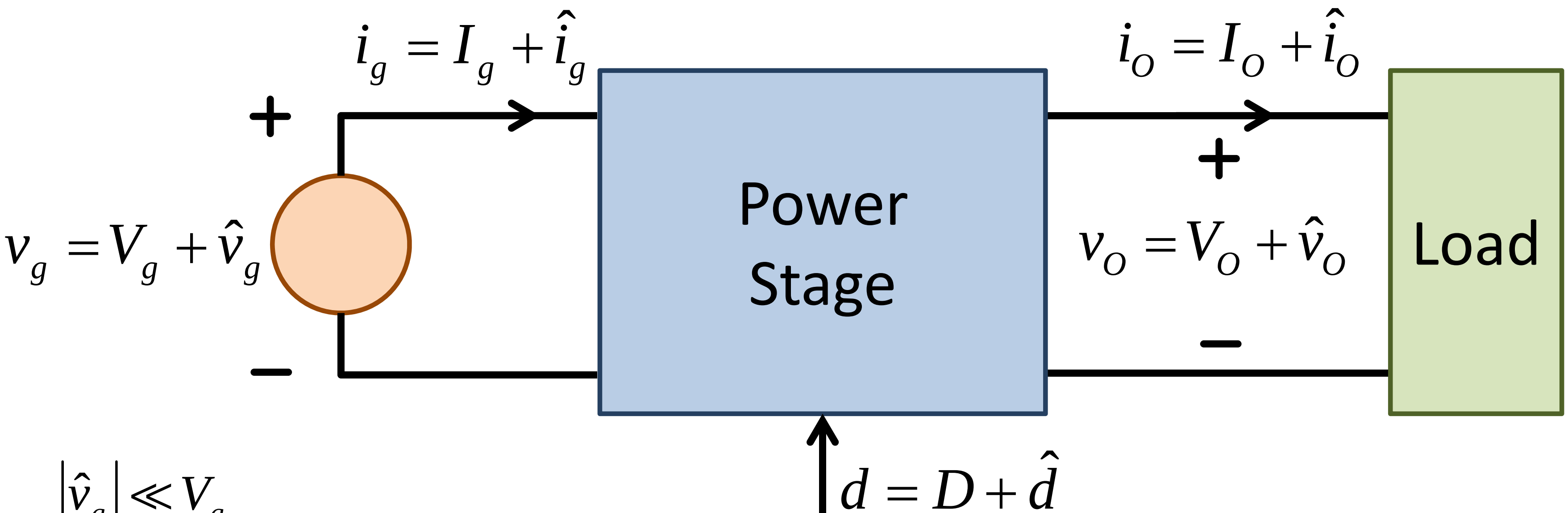
$$I_g = \frac{1}{T_S} \cdot I_L \cdot T_{ON} = \frac{1}{T_S} \cdot I_L \cdot D \cdot T_S = D \cdot I_L = D \cdot \frac{1}{R} \cdot V_C = D \cdot I_{OUT}$$

Buck Converter DC Model



EMBEDDED POWER LABS

AC (Small Signal) Modeling



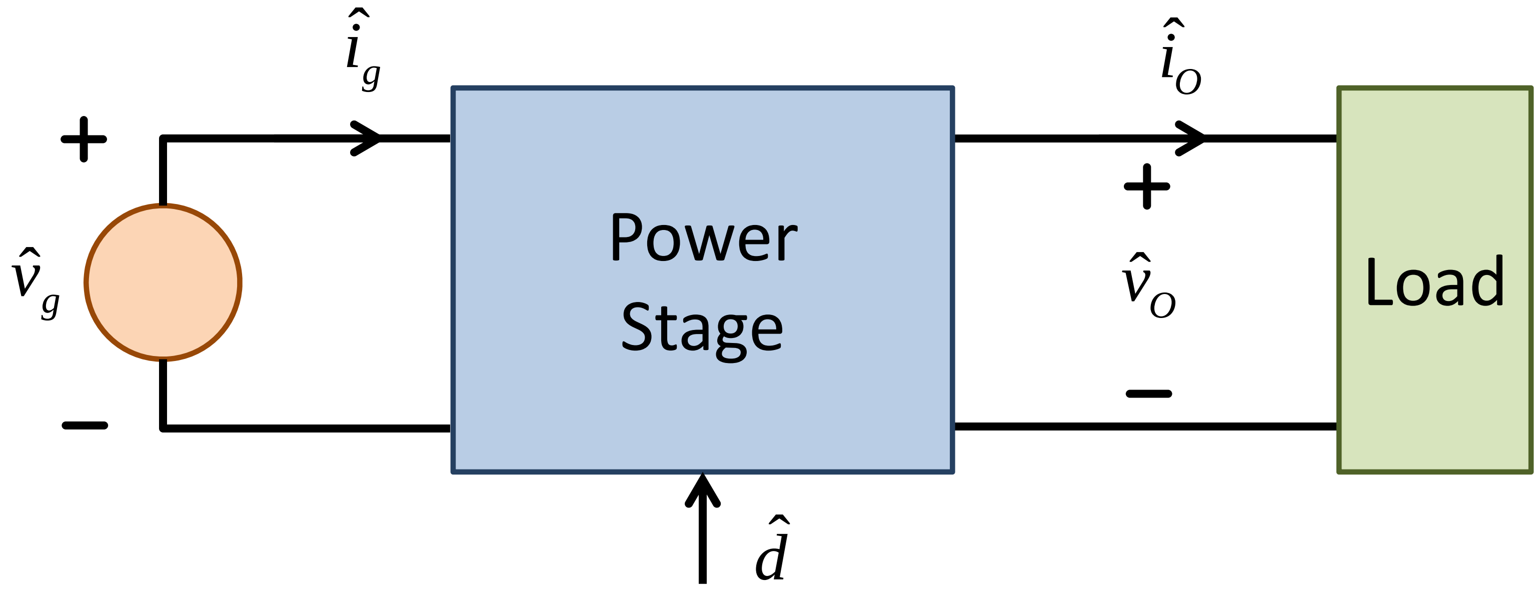
$$|\hat{v}_g| \ll V_g$$
$$|\hat{i}_g| \ll I_g$$
$$|\hat{v}_O| \ll V_O$$
$$|\hat{i}_O| \ll I_O$$
$$|\hat{d}| \ll D$$

Small Signal Modeling

How Small is Small? Small Enough The System Remains Linear

EMBEDDED POWER LABS

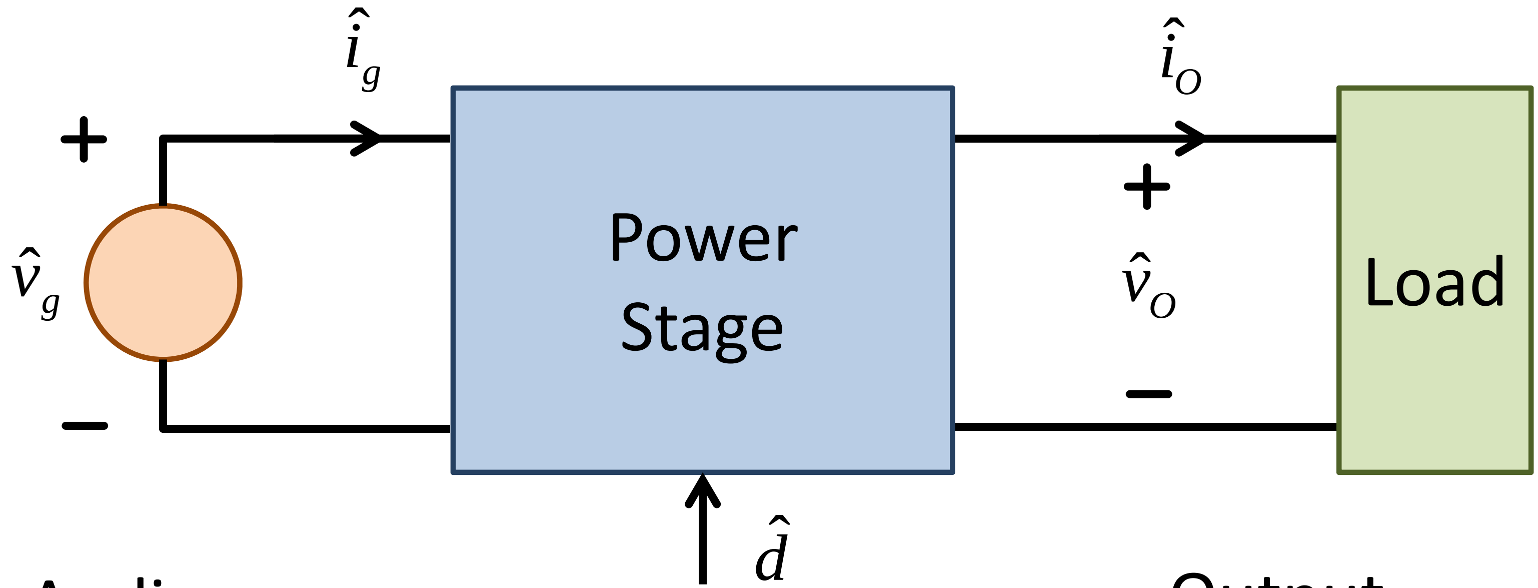
AC Modeling



Control To Output Transfer Function

$$G_{vd} = \left.\frac{\hat{v}_O}{\hat{d}}\right|_{\hat{v}_g=0}$$

AC Modeling



Audio-susceptibility

$$G_{vg} = \left. \frac{\hat{v}_O}{\hat{v}_g} \right|_{\hat{d}=0}$$

Input Impedance

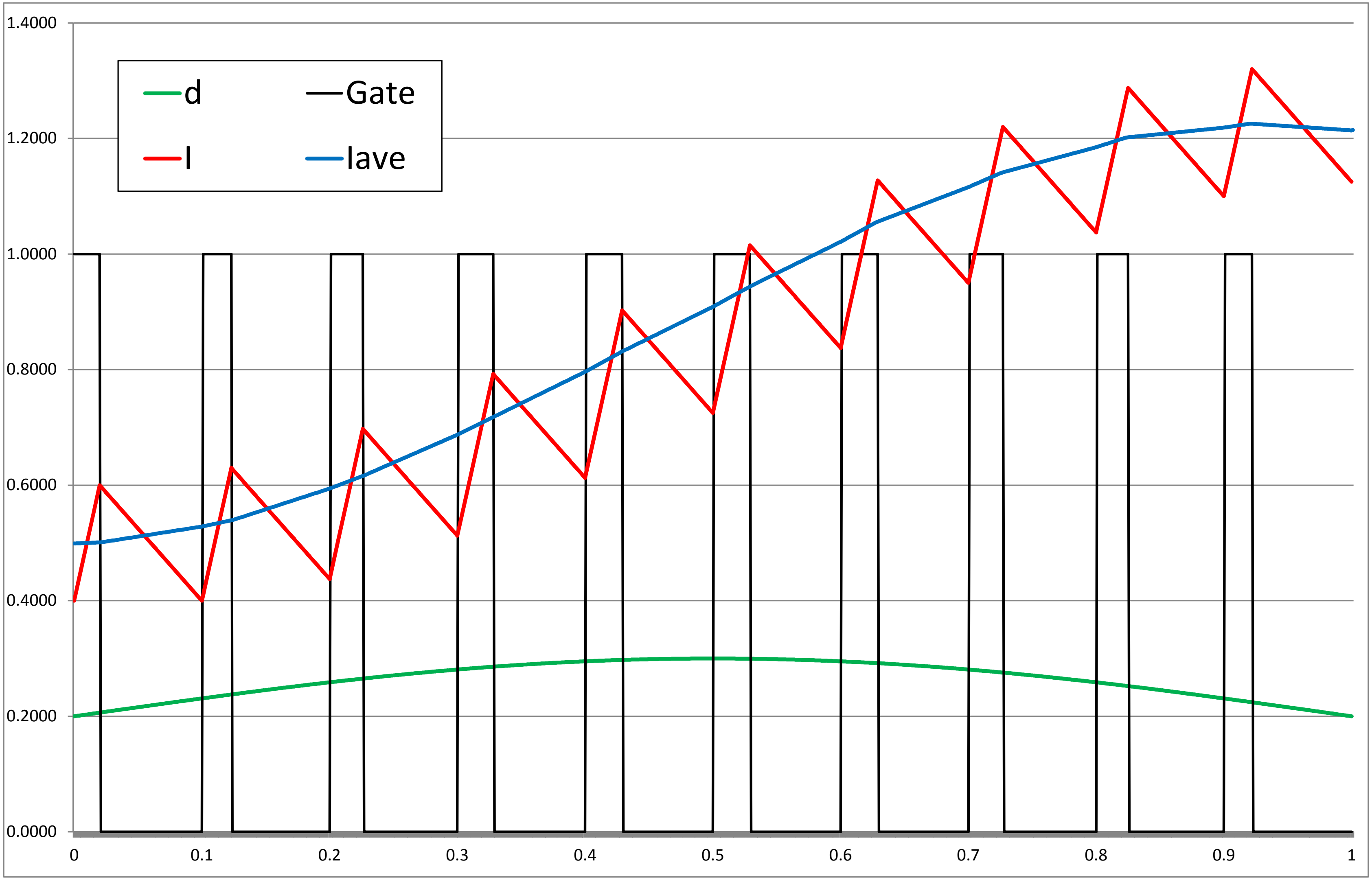
$$Z_i = \left. \frac{\hat{v}_g}{\hat{i}_g} \right|_{\hat{d}=0}$$

Output Impedance

$$Z_O = \left. \frac{\hat{v}_O}{\hat{i}_O} \right|_{\hat{d}=0, \hat{v}_g=0}$$

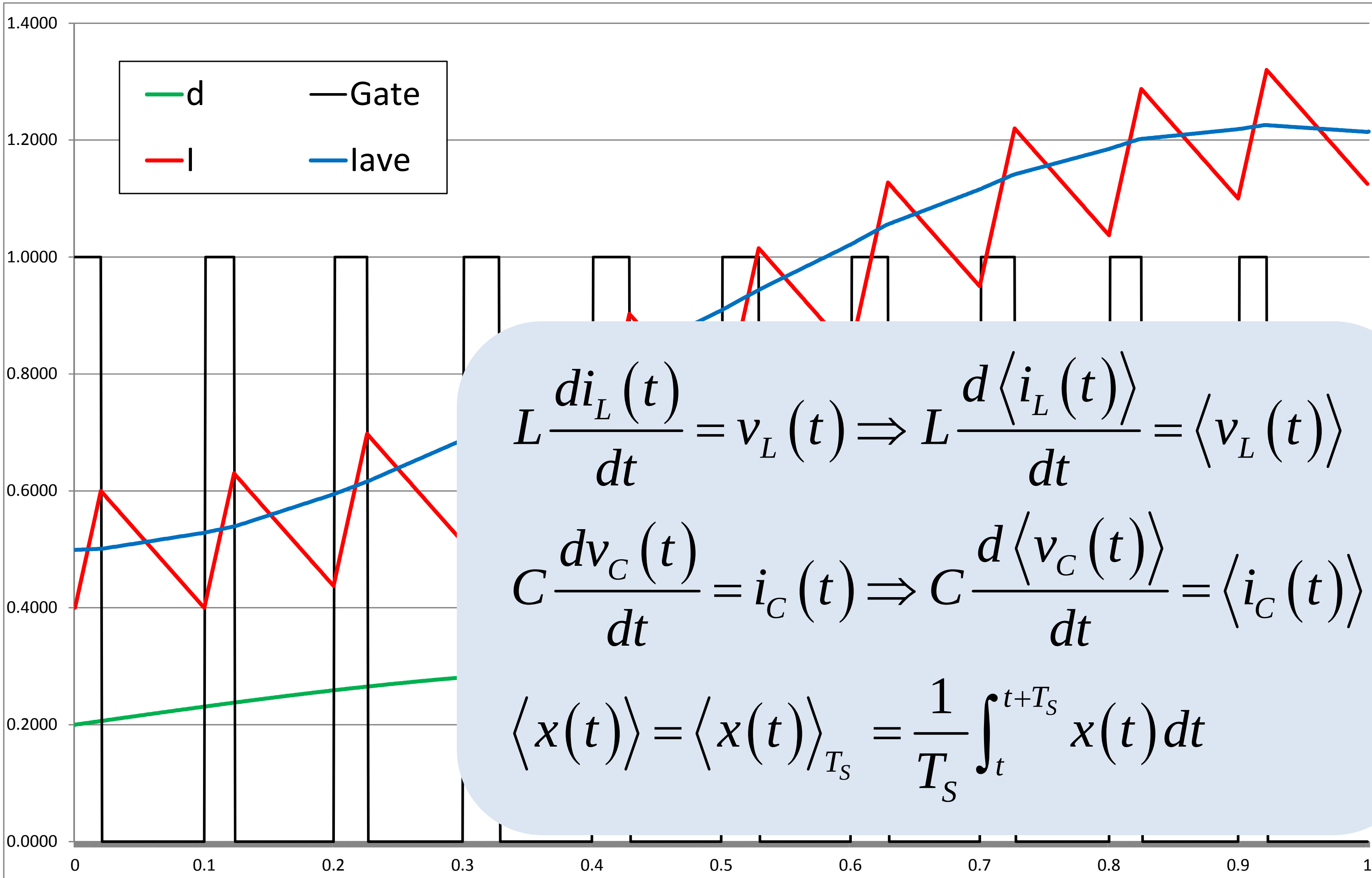
EMBEDDED POWER LABS

Averaging



d
Gate
I
Iave
1.4000
1.2000
1.0000
0.8000
0.6000
0.4000
0.2000
0.0000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Averaging



$$L\frac{di_L(t)}{dt} = v_L(t) \Rightarrow L\frac{d\langle i_L(t)\rangle}{dt} = \langle v_L(t)\rangle$$

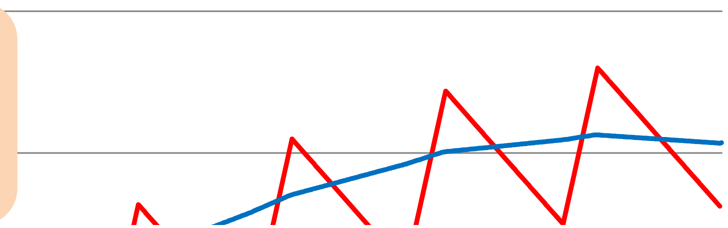
$$C\frac{dv_C(t)}{dt} = i_C(t) \Rightarrow C\frac{d\langle v_C(t)\rangle}{dt} = \langle i_C(t)\rangle$$

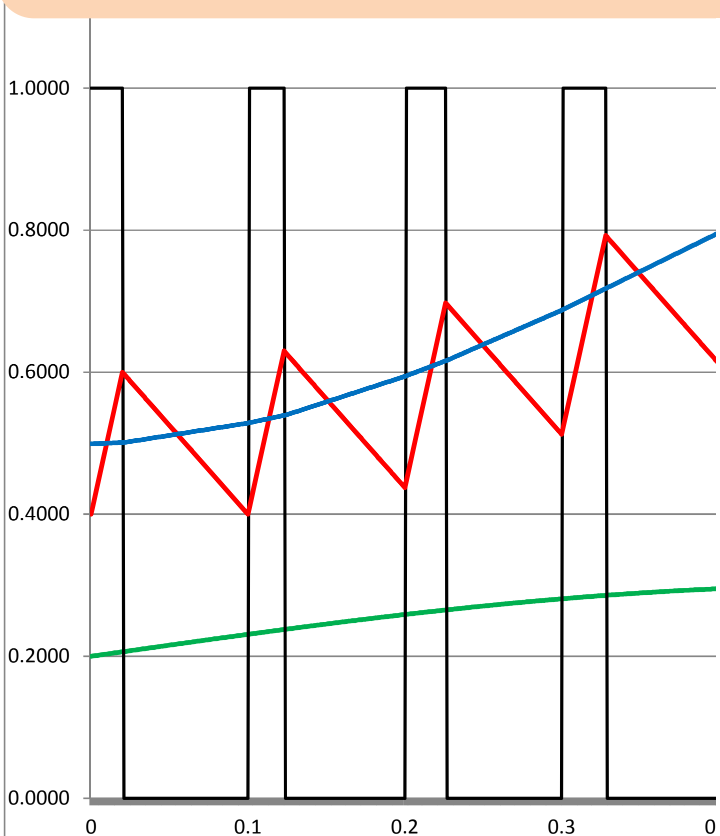
$$\langle x(t)\rangle = \langle x(t)\rangle_{T_S} = \frac{1}{T_S}\int_t^{t+T_S} x(t)\,dt$$

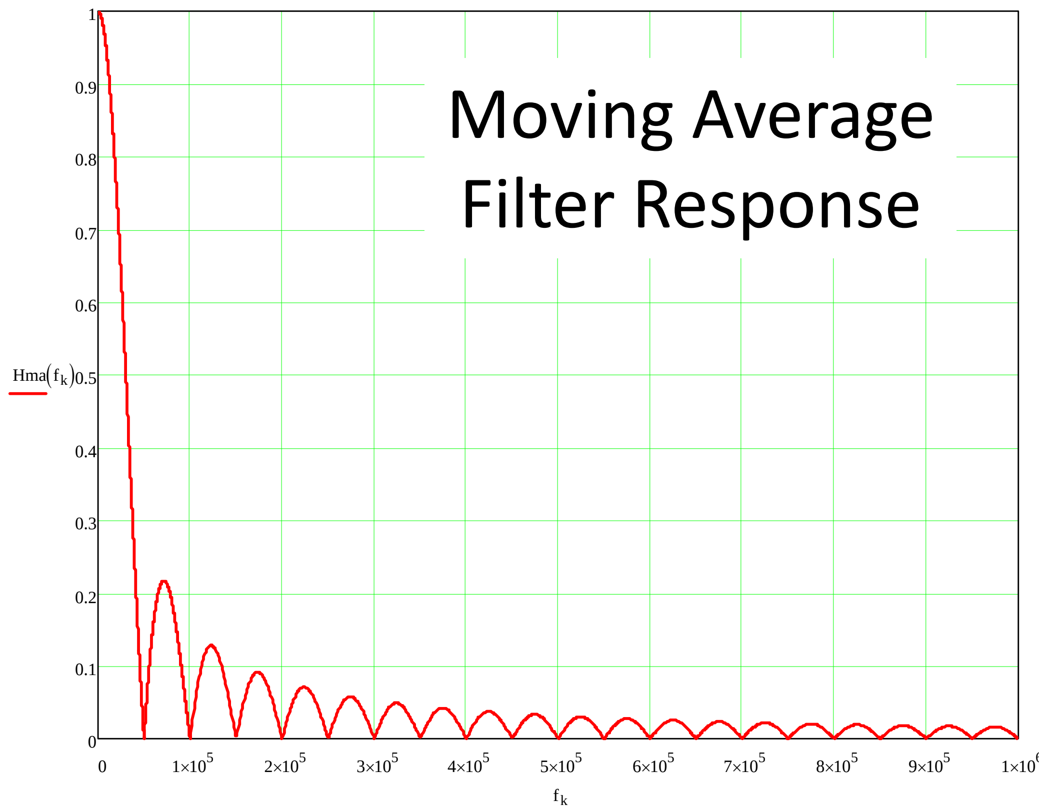
EMBEDDED POWER LABS

Another View Of Averaging

$$\langle x(t)\rangle = \langle x(t)\rangle_{T_S} = \frac{1}{T_S}\cdot\int_{t}^{t+T_S} x(t)\,dt$$







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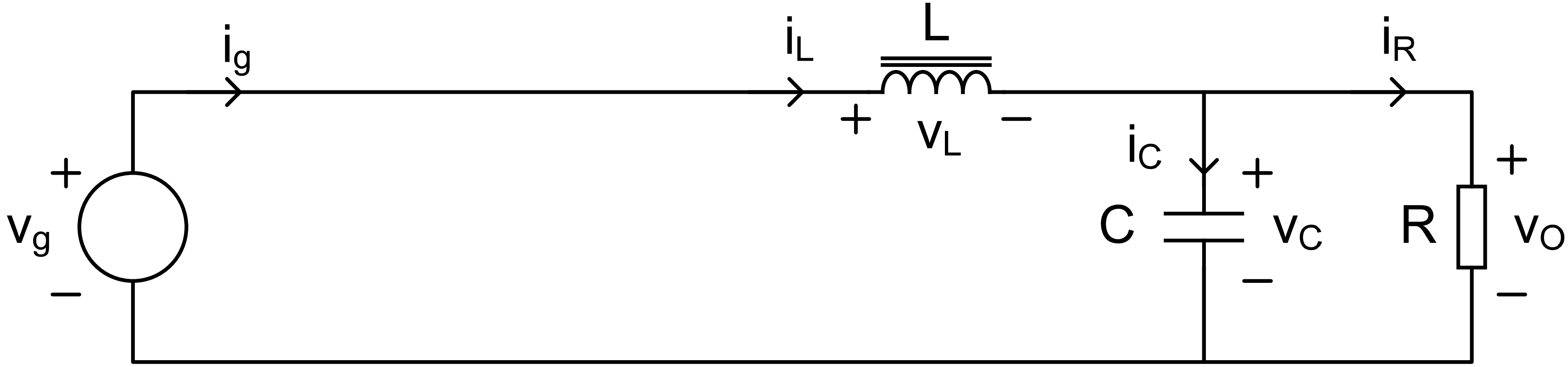
Modeling The Buck Converter: On Time



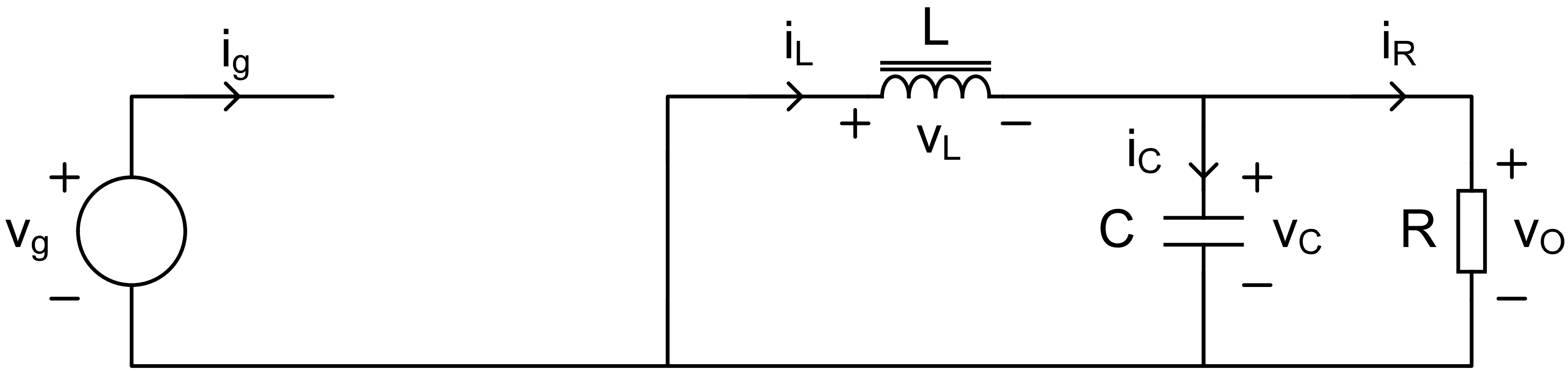
$$v_L(t) = L\frac{di_L(t)}{dt} = v_g(t) - v_O(t) \approx \langle v_g(t)\rangle - \langle v_C(t)\rangle$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = i_L(t) - \frac{1}{R}v_C(t) \approx \langle i_L(t)\rangle - \frac{1}{R}\langle v_C(t)\rangle$$

Small Ripple Approximation

EMBEDDED POWER LABS

Modeling The Buck Converter: Off Time



$$v_L(t) = L\frac{di_L(t)}{dt} = -v_C(t) \approx -\langle v_C(t)\rangle$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = i_L(t) - \frac{1}{R}v_C(t) \approx \langle i_L(t)\rangle - \frac{1}{R}\langle v_C(t)\rangle$$

Averaging The Inductor Voltage

$$\left\langle v_L(t)\right\rangle = \frac{1}{T_S}\int_t^{t+T_S} v_L(\tau)\,d\tau$$

$$\approx d(t)\cdot\left(\left\langle v_g(t)\right\rangle - \left\langle v_C(t)\right\rangle\right) + d'(t)\cdot\left(-\left\langle v_C(t)\right\rangle\right)$$

$$= d(t)\cdot\left(\left\langle v_g(t)\right\rangle - \left\langle v_C(t)\right\rangle\right) + \left(1-d(t)\right)\cdot\left(-\left\langle v_C(t)\right\rangle\right)$$

$$= d(t)\cdot\left\langle v_g(t)\right\rangle - d(t)\cdot\left\langle v_C(t)\right\rangle - \left\langle v_C(t)\right\rangle + d(t)\cdot\left\langle v_C(t)\right\rangle$$

$$= d(t)\cdot\left\langle v_g(t)\right\rangle - \left\langle v_C(t)\right\rangle$$

$$\left\langle v_L(t)\right\rangle = L\frac{d\left\langle i_L(t)\right\rangle}{dt} = d(t)\cdot\left\langle v_g(t)\right\rangle - \left\langle v_C(t)\right\rangle$$

Averaging The Capacitor Current

$$\langle i_C(t)\rangle = d(t)\cdot\left(\langle i_L(t)\rangle - \frac{1}{R}\cdot\langle v_C(t)\rangle\right) + d'(t)\cdot\left(\langle i_L(t)\rangle - \frac{1}{R}\cdot\langle v_C(t)\rangle\right)$$

$$= \langle i_L(t)\rangle - \frac{1}{R}\cdot\langle v_C(t)\rangle$$

$$\langle i_C(t)\rangle = C\frac{d\langle v_C(t)\rangle}{dt} = \langle i_L(t)\rangle - \frac{1}{R}\cdot\langle v_C(t)\rangle$$

Averaging The Input Current

$$\langle i_g(t)\rangle = d(t)\cdot\langle i_L(t)\rangle + d'(t)\cdot 0$$

$$= d(t)\cdot\langle i_L(t)\rangle$$

Perturb And Linearize

DC Operating Point

$$V_g \qquad V_C = D \cdot V_g$$

$$I_L = \frac{1}{R} \cdot V_C \qquad I_g = D \cdot I_L$$

Input Perturbation

$$\langle v_g(t) \rangle = V_g + \hat{v}_g(t)$$

$$d(t) = D + \hat{d}(t)$$

Resulting Output Perturbation

$$\langle i_L(t) \rangle = I_L + \hat{i}_L(t)$$

$$\langle v_C(t) \rangle = V_C + \hat{v}_C(t)$$

$$\langle i_g(t) \rangle = I_g + \hat{i}_g(t)$$

Small Signal Constraint

$$\left|\hat{v}_g(t)\right| \ll \left|V_g\right|$$

$$\left|\hat{d}(t)\right| \ll \left|D\right|$$

$$\left|\hat{i}_L(t)\right| \ll \left|I_L\right|$$

$$\left|\hat{v}_C(t)\right| \ll \left|V_C\right|$$

$$\left|\hat{i}_g(t)\right| \ll \left|I_g\right|$$

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Perturb And Linearize

$$L\frac{d\left\langle i_L(t)\right\rangle}{dt} = d(t)\cdot\left\langle v_g t\right\rangle - \left\langle v_C(t)\right\rangle$$ Averaged Differential Equation

Substitute DC Plus Perturbation For Average Values:

$$L\frac{d\left(I_L + \hat{i}_L(t)\right)}{dt} = \left(D + \hat{d}(t)\right)\cdot\left(V_g + \hat{v}_g(t)\right) - \left(V_C + \hat{v}_C(t)\right)$$

Expand:

$$L\frac{dI_L}{dt} + L\frac{d\hat{i}_L(t)}{dt} = D\cdot V_g + D\cdot\hat{v}_g(t) + V_g\cdot\hat{d}(t) + \hat{d}(t)\cdot\hat{v}_g(t) - V_C - \hat{v}_C(t)$$

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Perturb And Linearize

Collect Terms:

$$L\frac{dI_L}{dt} + L\frac{d\hat{i}_L(t)}{dt} = D\cdot V_g - V_C$$

$$+D\cdot\hat{v}_g(t) + V_g\cdot\hat{d}(t) - \hat{v}_C(t)$$

$$+\hat{d}(t)\cdot\hat{v}_g(t)$$

DC Terms Equal Zero

Discard 2nd Order Terms

This Leaves A First Order Equation:

$$L\frac{d\hat{i}_L(t)}{dt} = +D\cdot\hat{v}_g(t) + V_g\cdot\hat{d}(t) - \hat{v}_C(t)$$

Perturb And Linearize

$$C\frac{d\langle v_C(t)\rangle}{dt}=\langle i_L(t)\rangle-\frac{1}{R}\cdot\langle v_C(t)\rangle$$ Averaged Differential Equation

Substitute DC Plus Perturbation For Average Values:

$$C\frac{d\left(V_C+\hat{v}_C(t)\right)}{dt}=I_L+\hat{i}_L(t)-\frac{1}{R}\cdot\left(V_C+\hat{v}_C(t)\right)$$

Expand:

$$C\frac{dV_C}{dt}+C\frac{d\hat{v}_C(t)}{dt}=I_L-\frac{1}{R}\cdot V_C+\hat{i}_L(t)-\frac{1}{R}\cdot\hat{v}_C(t)$$

DC Terms Equal Zero

$$C\frac{d\hat{v}_C(t)}{dt}=\hat{i}_L(t)-\frac{1}{R}\cdot\hat{v}_C(t)$$

Perturb And Linearize

$$\left\langle i_g(t)\right\rangle = d(t)\cdot\left\langle i_L(t)\right\rangle$$

$$I_g + \hat{i}_g(t) = \left(D + \hat{d}(t)\right)\cdot\left(I_L + \hat{i}_L(t)\right)$$

$$= D\cdot I_L + D\cdot\hat{i}_L(t) + I_L\cdot\hat{d}(t) + \hat{d}(t)\cdot\hat{i}_L(t)$$

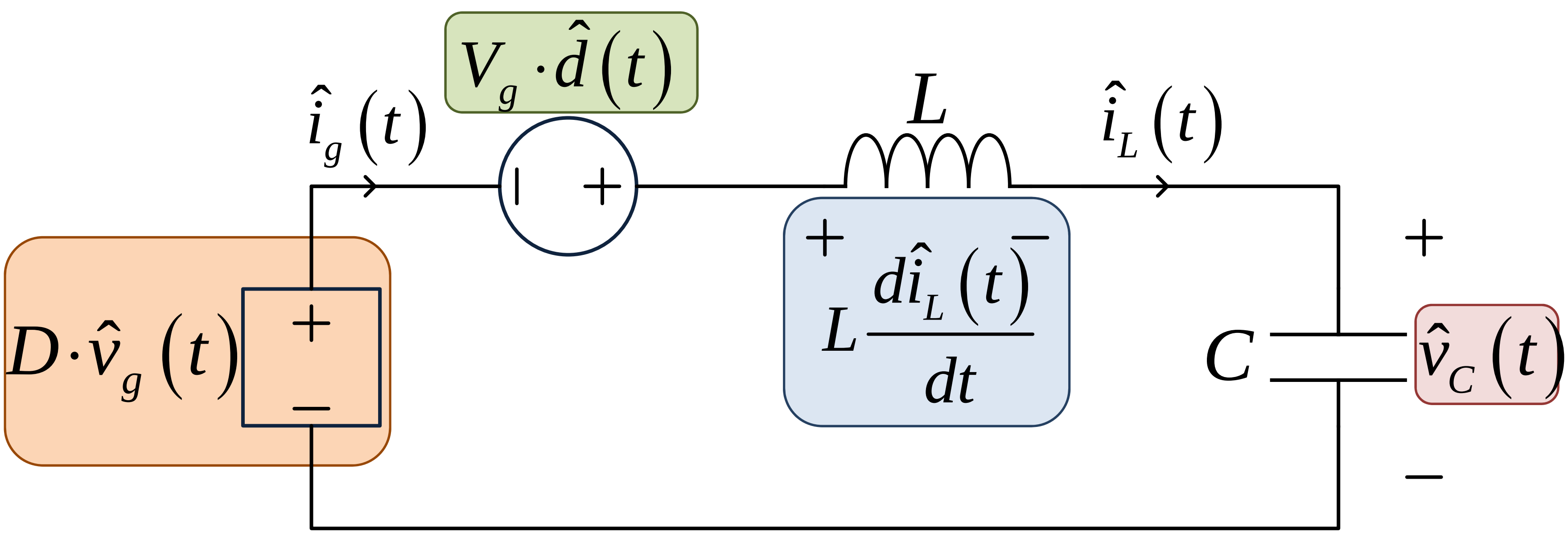
$$\hat{i}_g(t) = D\cdot\hat{i}_L(t) + I_L\cdot\hat{d}(t) + \cancel{\hat{d}(t)\cdot\hat{i}_L(t)}$$

$$\hat{i}_g(t) = D\cdot\hat{i}_L(t) + I_L\cdot\hat{d}(t)$$

EMBEDDED POWER LABS

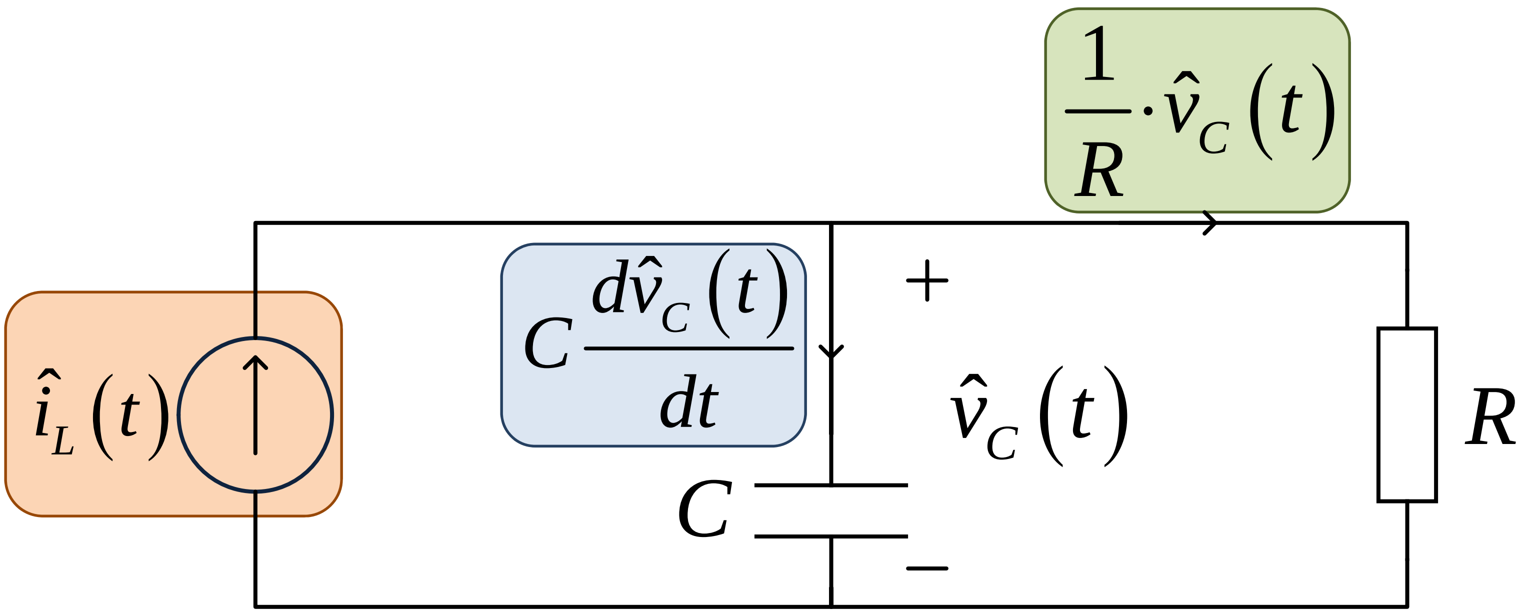
Construct The Model

$$L\frac{d\hat{i}_L(t)}{dt} = +D\cdot\hat{v}_g(t) + V_g\cdot\hat{d}(t) - \hat{v}_C(t)$$



Construct The Model

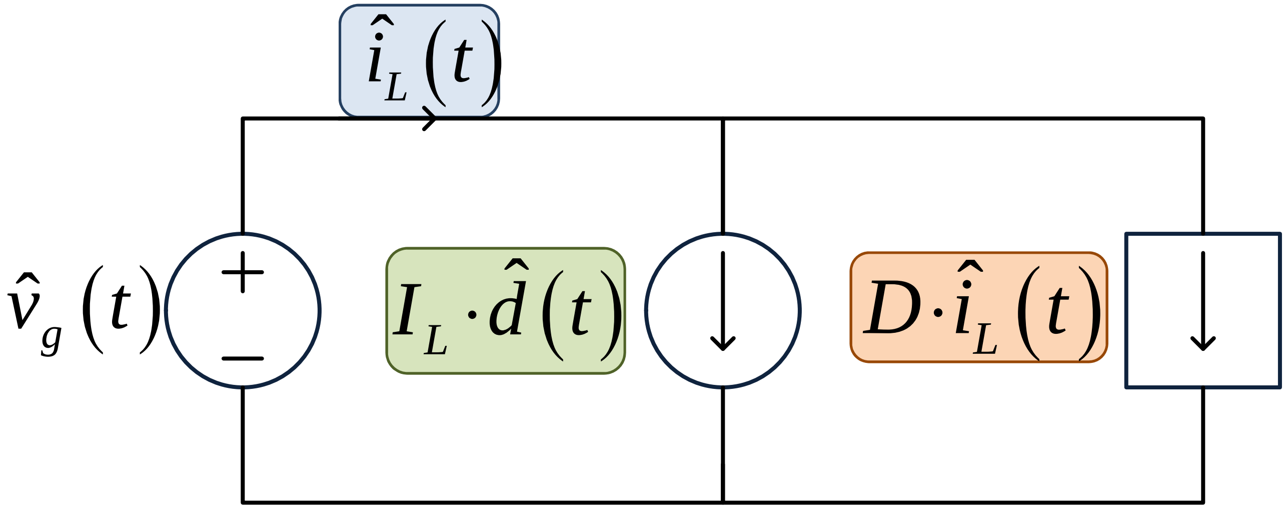
$$C\frac{d\hat{v}_C(t)}{dt} = \hat{i}_L(t) - \frac{1}{R}\cdot\hat{v}_C(t)$$



EMBEDDED POWER LABS

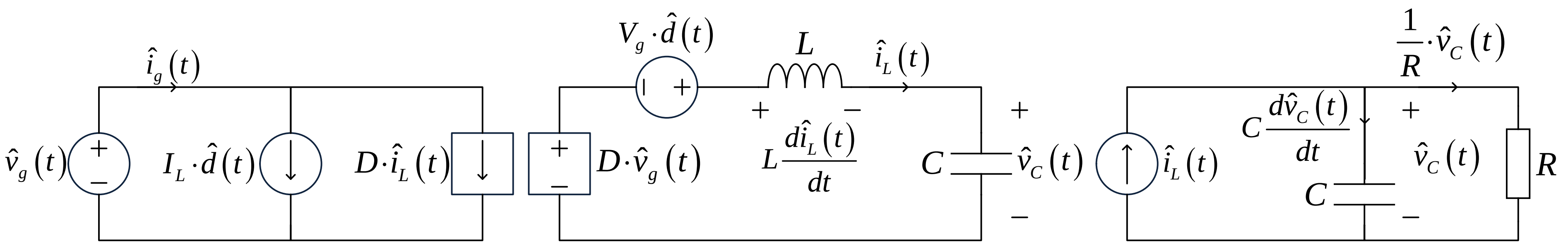
Construct The Model

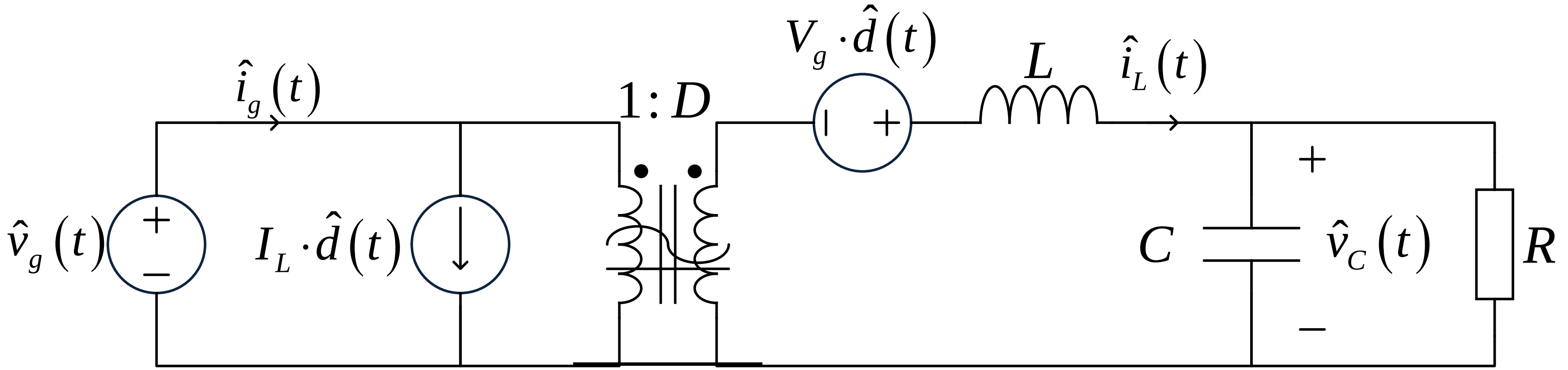
$$\hat{i}_g(t) = D \cdot \hat{i}_L(t) + I_L \cdot \hat{d}(t)$$



EMBEDDED POWER LABS

Construct The Model

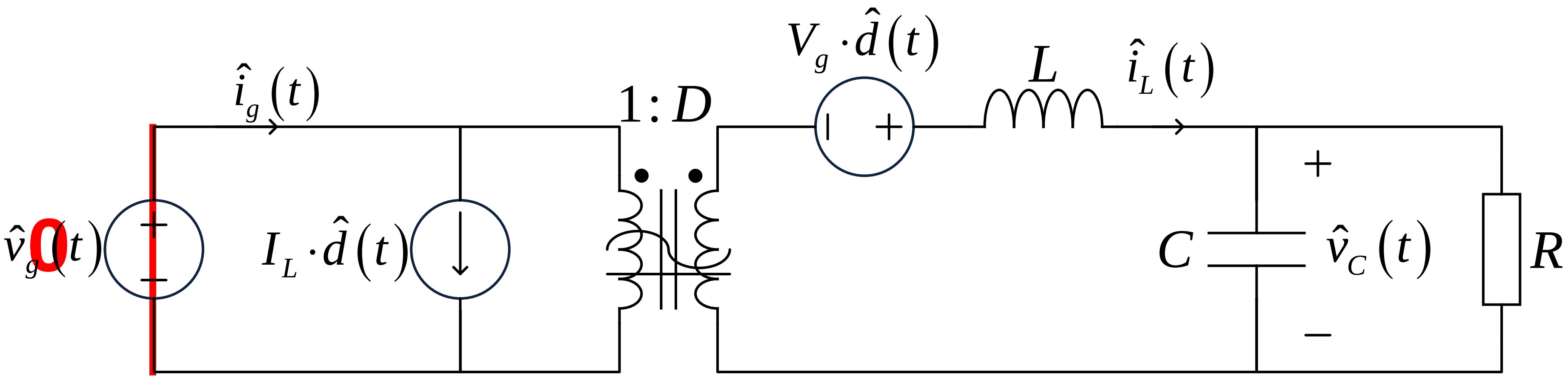




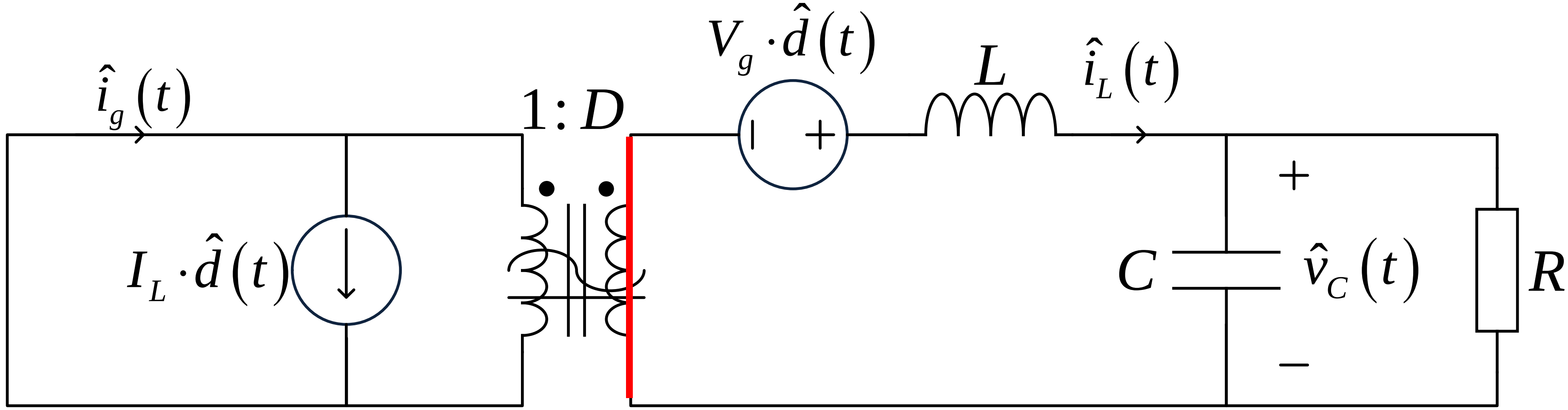
EMBEDDED POWER LABS

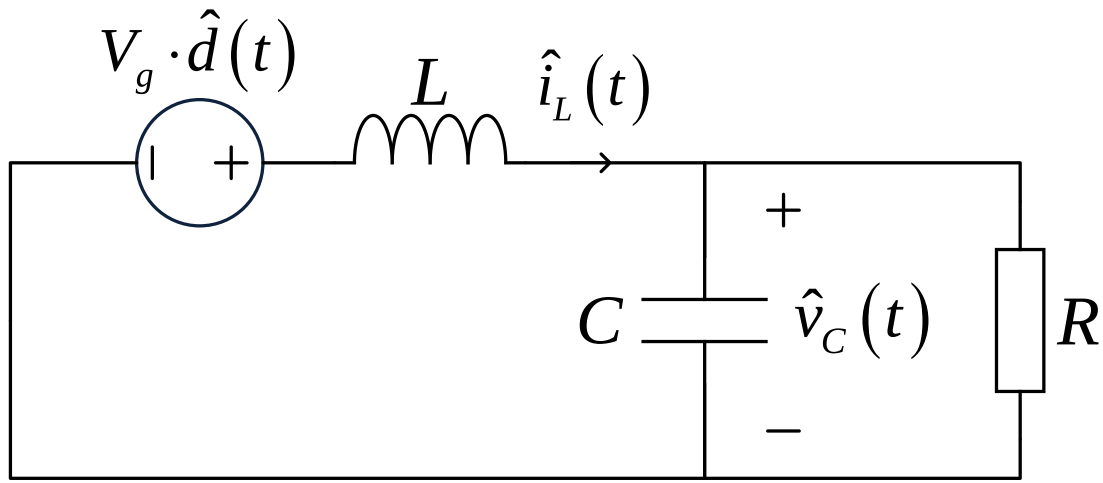
Control-To-Output Transfer Function

$$G_{vd}(s) = \left.\frac{\hat{v}_C(s)}{\hat{d}(s)}\right|_{\hat{v}_g=0}$$



Control-To-Output Transfer Function

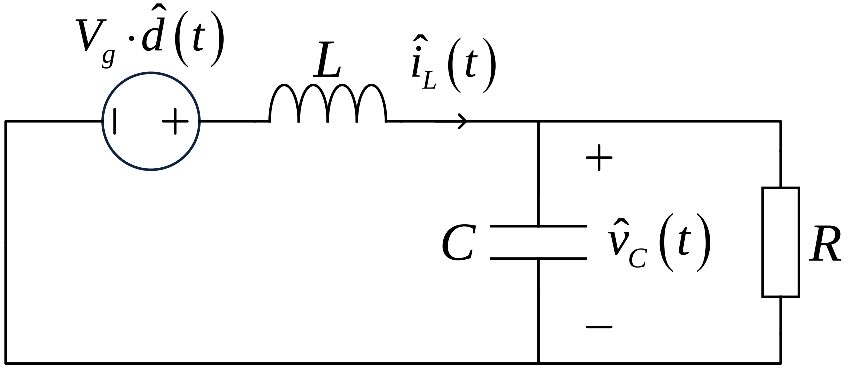




EMBEDDED POWER LABS

Control-To-Output Transfer Function

$$\hat{v}_C(s) = \frac{R \| \frac{1}{s \cdot C}}{s \cdot L + R \| \frac{1}{s \cdot C}} \cdot V_g \cdot \hat{d}(s)$$



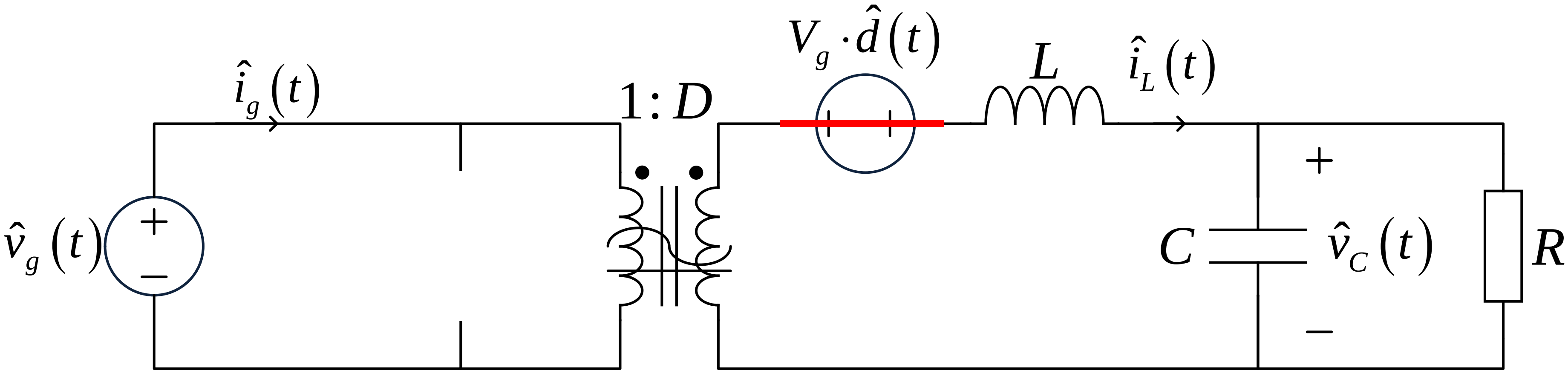
$$G_{vd}(s) = \frac{\hat{v}_C(s)}{\hat{d}(s)} = \frac{1}{1 + \frac{L}{R} \cdot s + L \cdot C \cdot s^2} \cdot V_g = \frac{1}{1 + \frac{1}{Q} \cdot \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \cdot V_g$$

$$\omega_0^2 = \frac{1}{L \cdot C} \qquad Q = R \cdot \sqrt{\frac{C}{L}} = \frac{1}{2 \cdot \zeta}$$

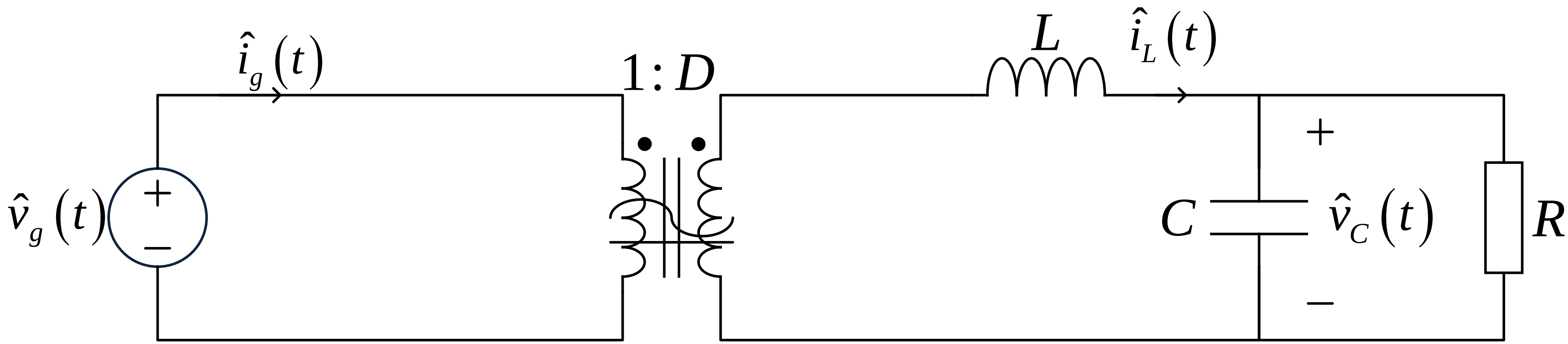
Input To Output Transfer Function

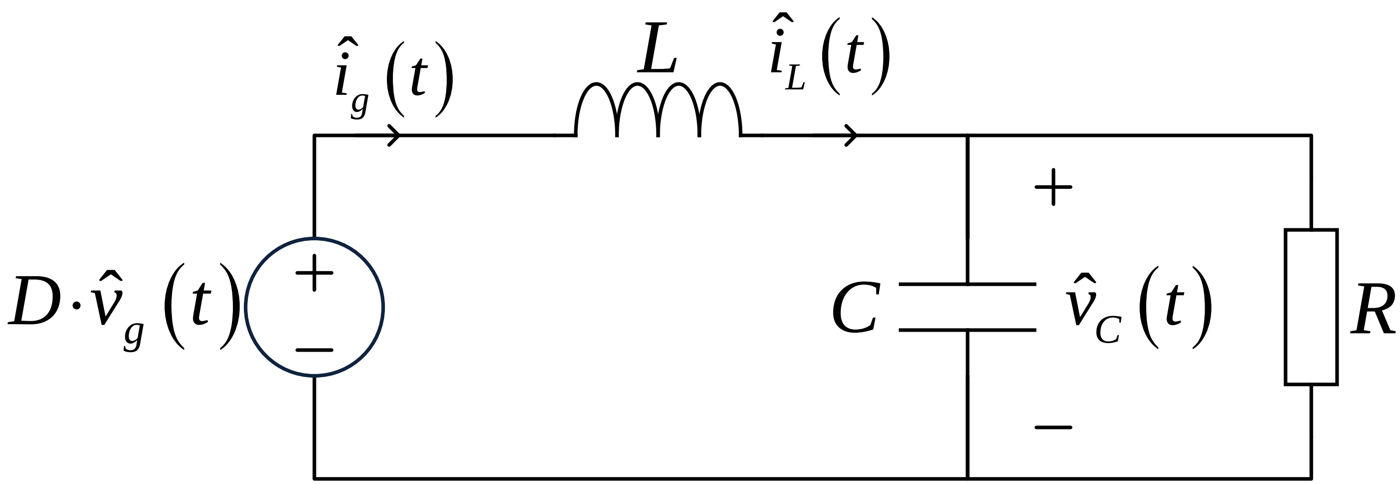
$$G_{vg}(s) = \left.\frac{\hat{v}_C(s)}{\hat{v}_g(s)}\right|_{\hat{d}=0}$$

"Audiosusceptibility"



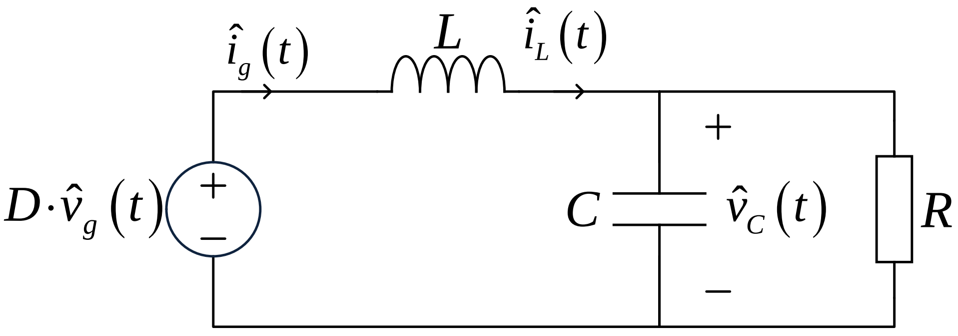
Input To Output Transfer Function





Input To Output Transfer Function

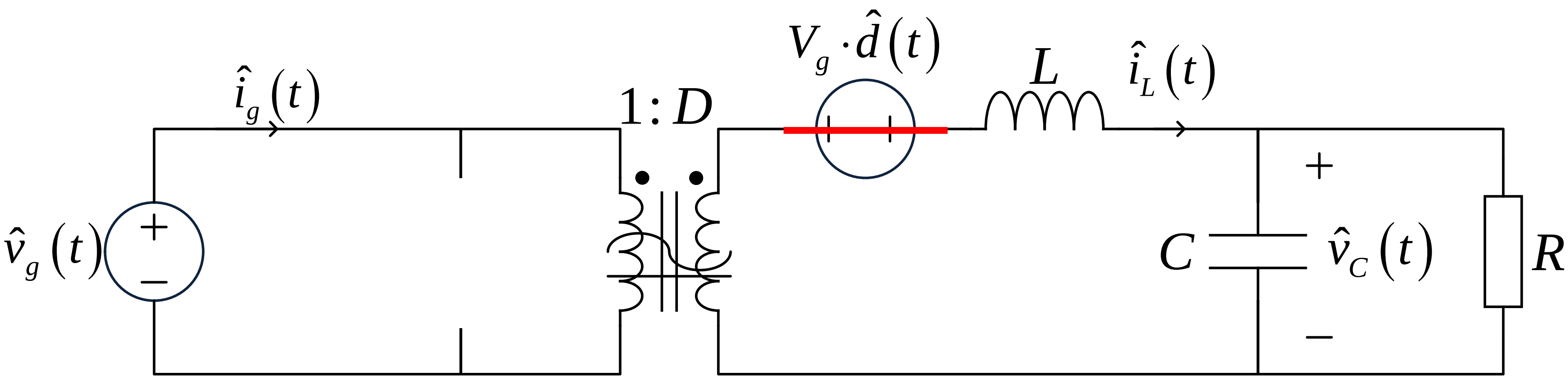
$$\hat{v}_C(s) = \frac{R \parallel \frac{1}{s \cdot C}}{s \cdot L + R \parallel \frac{1}{s \cdot C}} \cdot D \cdot \hat{v}_g$$



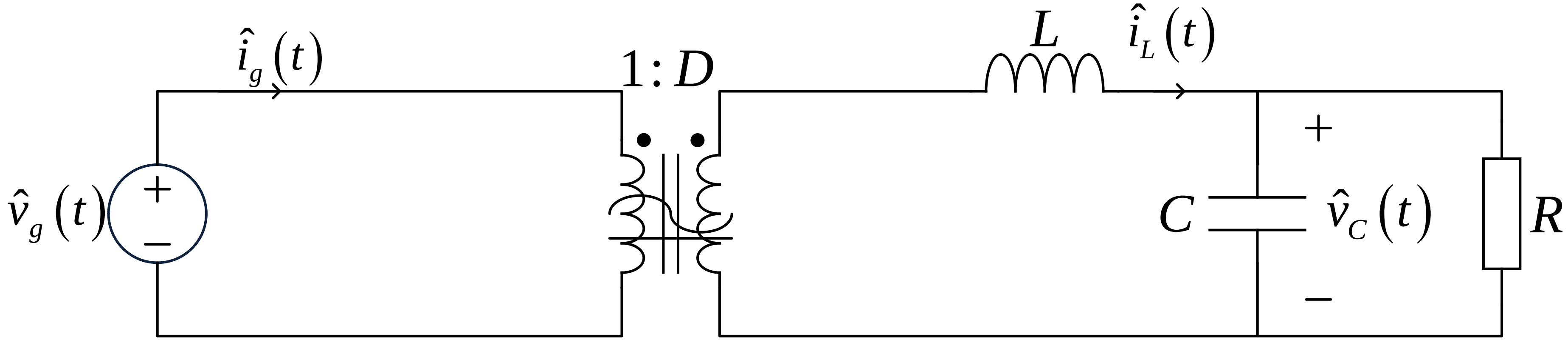
$$G_{vd}(s) = \frac{\hat{v}_C(s)}{\hat{v}_g} = \frac{D}{1 + \frac{L}{R} \cdot s + L \cdot C \cdot s^2} = \frac{D}{1 + \frac{1}{Q} \cdot \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

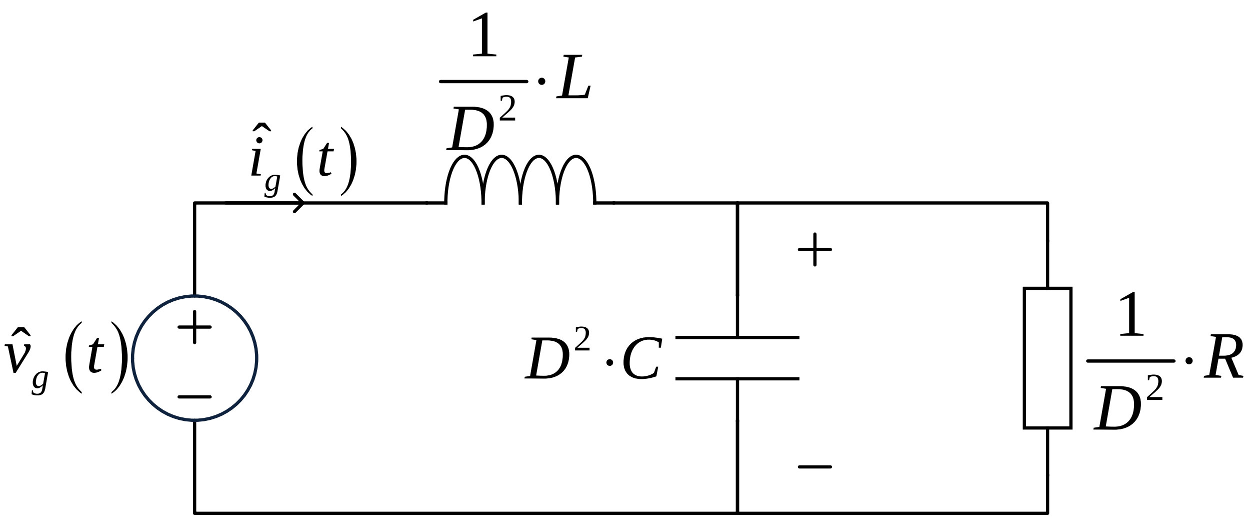
Input Impedance

$$Z_i(s) = \left.\frac{\hat{v}_g(s)}{\hat{i}_g(s)}\right|_{\hat{d}=0}$$



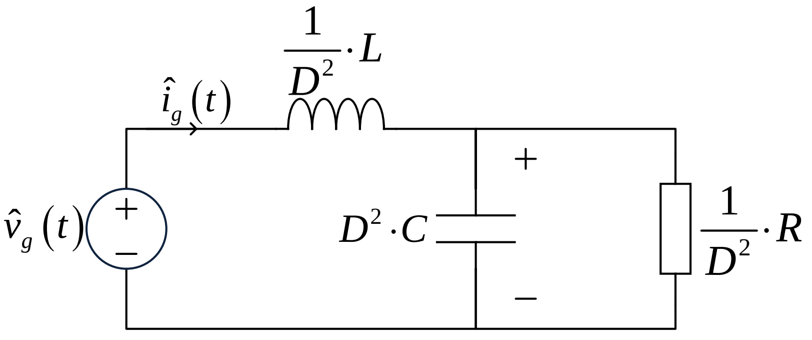
Input Impedance





Input Impedance

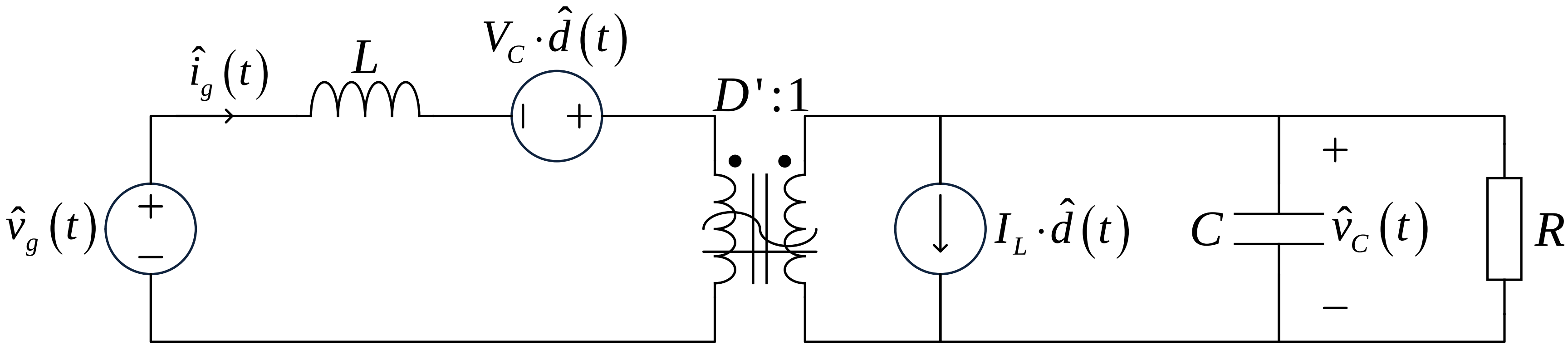
$$\hat{i}_g(s) = \frac{\hat{v}_g(s)}{s\cdot\frac{L}{D^2} + \frac{R}{D^2} \parallel \frac{1}{s\cdot D^2\cdot C}}$$

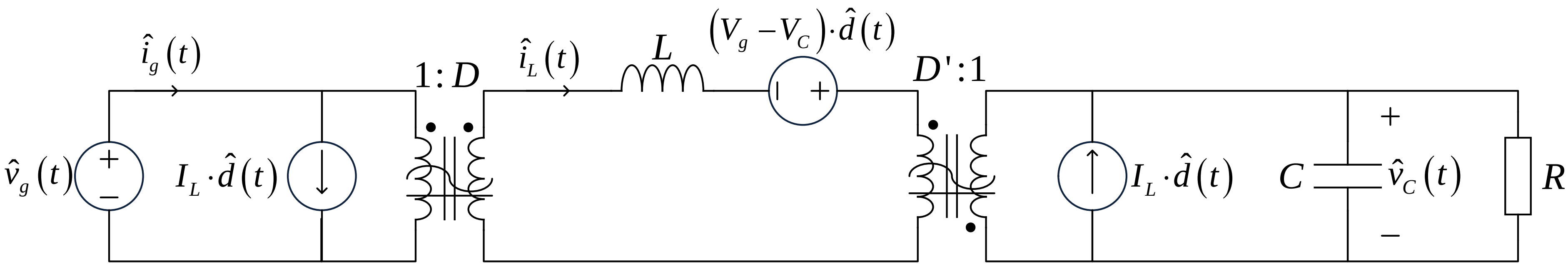


$$Z_i(s) = \frac{\hat{v}_g(s)}{\hat{i}_g(s)} = s\cdot\frac{L}{D^2} + \frac{R}{D^2} \parallel \frac{1}{s\cdot D^2\cdot C}$$

$$Z_i(s) = \frac{R}{D^2}\cdot\frac{1+s\cdot\frac{L}{R}+s^2\cdot L\cdot C}{1+s\cdot R\cdot C} = \frac{R}{D^2}\cdot\frac{1+\frac{1}{Q}\cdot\frac{s}{\omega_0}+\left(\frac{s}{\omega_0}\right)^2}{1+s\cdot R\cdot C}$$

Boost And Buck-Boost Models

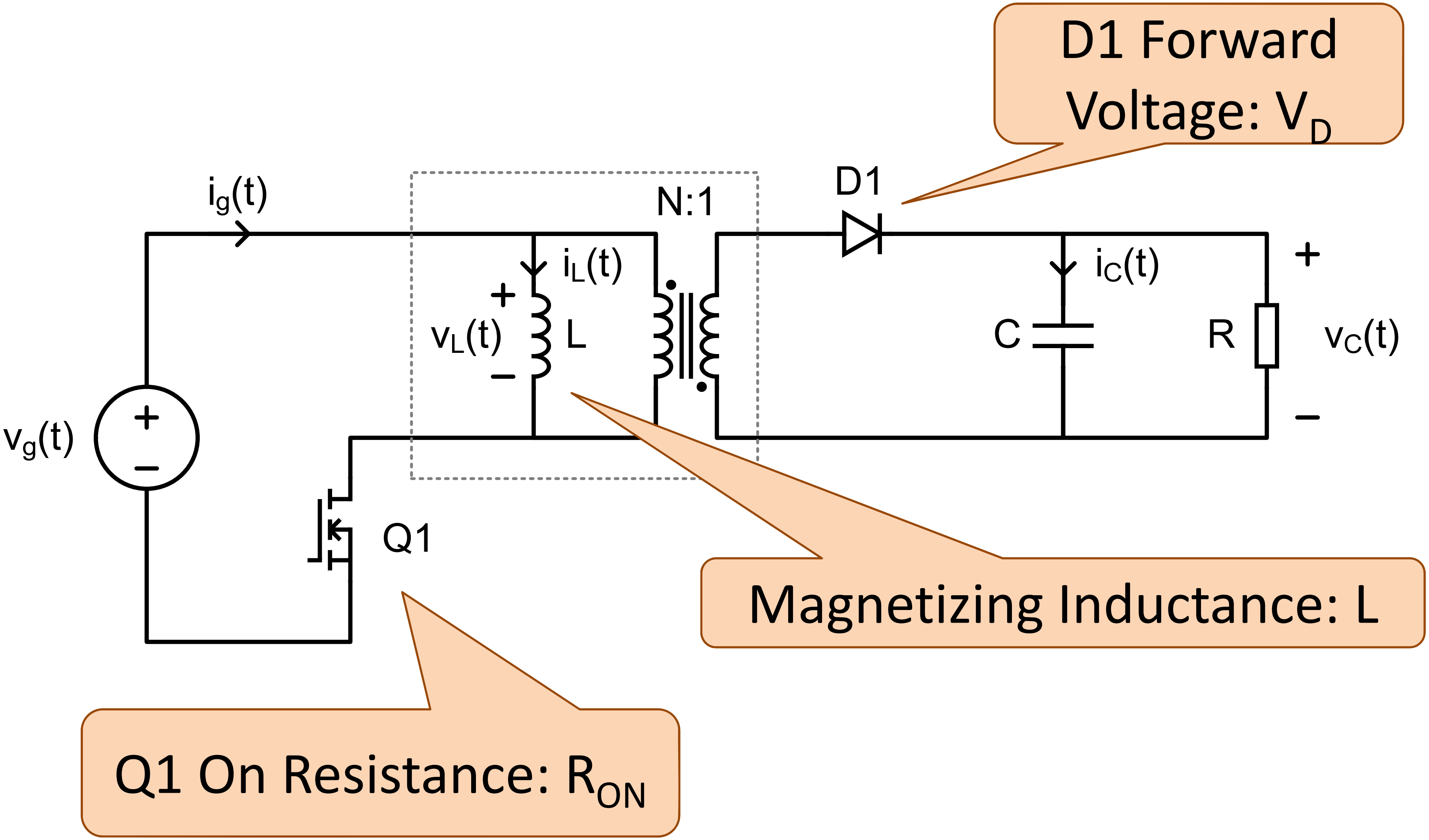




Re-drawn from "Fundamentals of Power Electronics", 2^{nd} ed., Erickson and Maksimovic, Figure 7.17

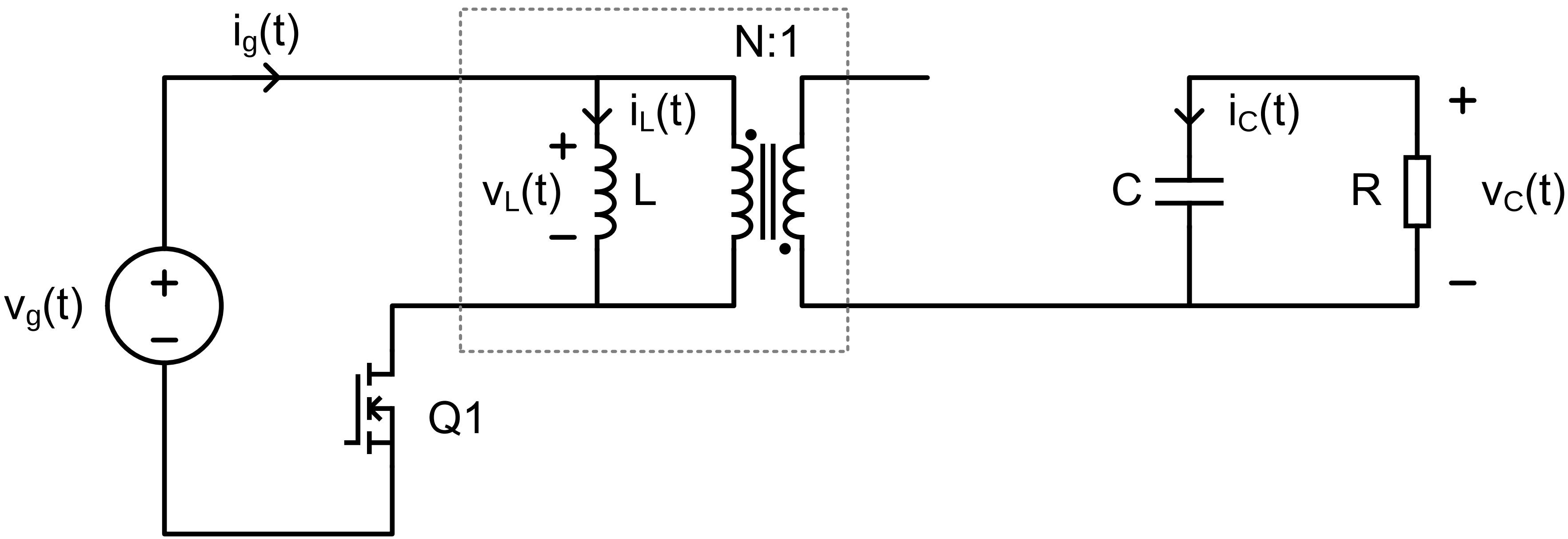
EMBEDDED POWER LABS

Flyback Example



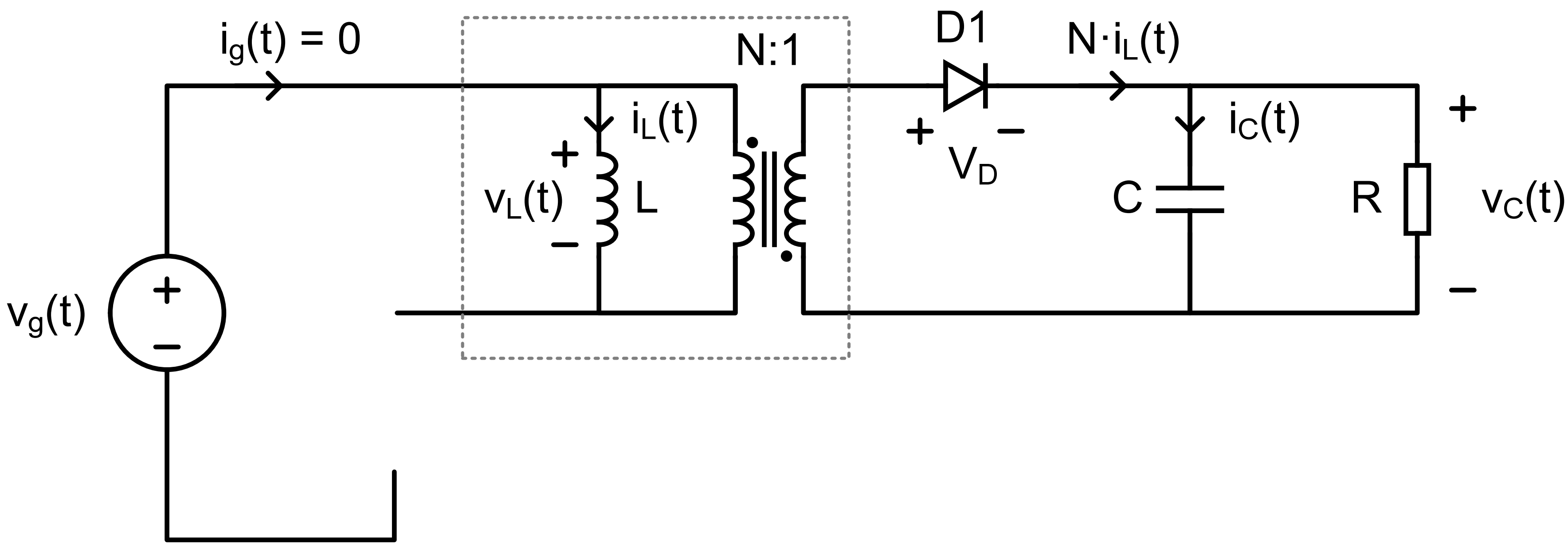
EMBEDDED POWER LABS

On-Time Circuit



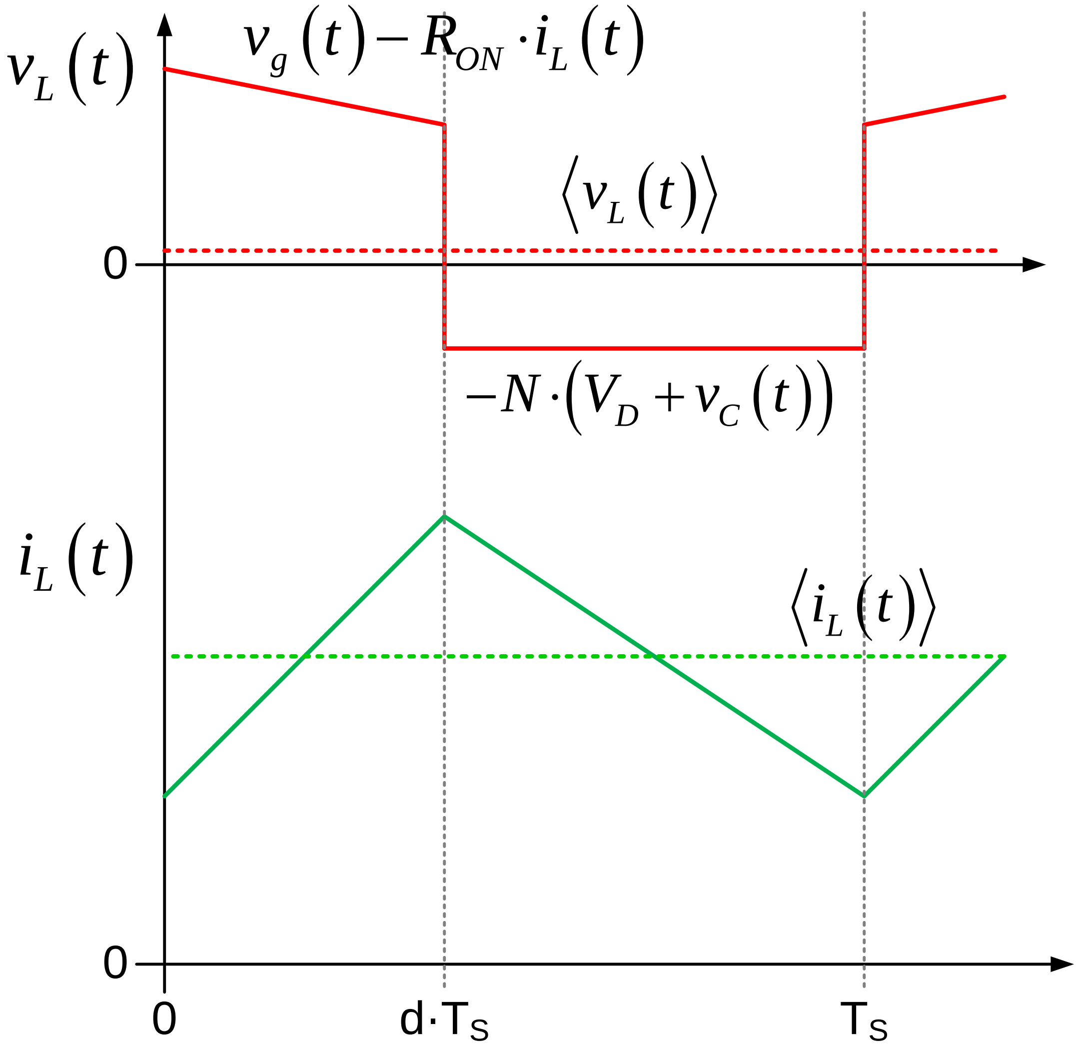
EMBEDDED POWER LABS

Off Time Circuit



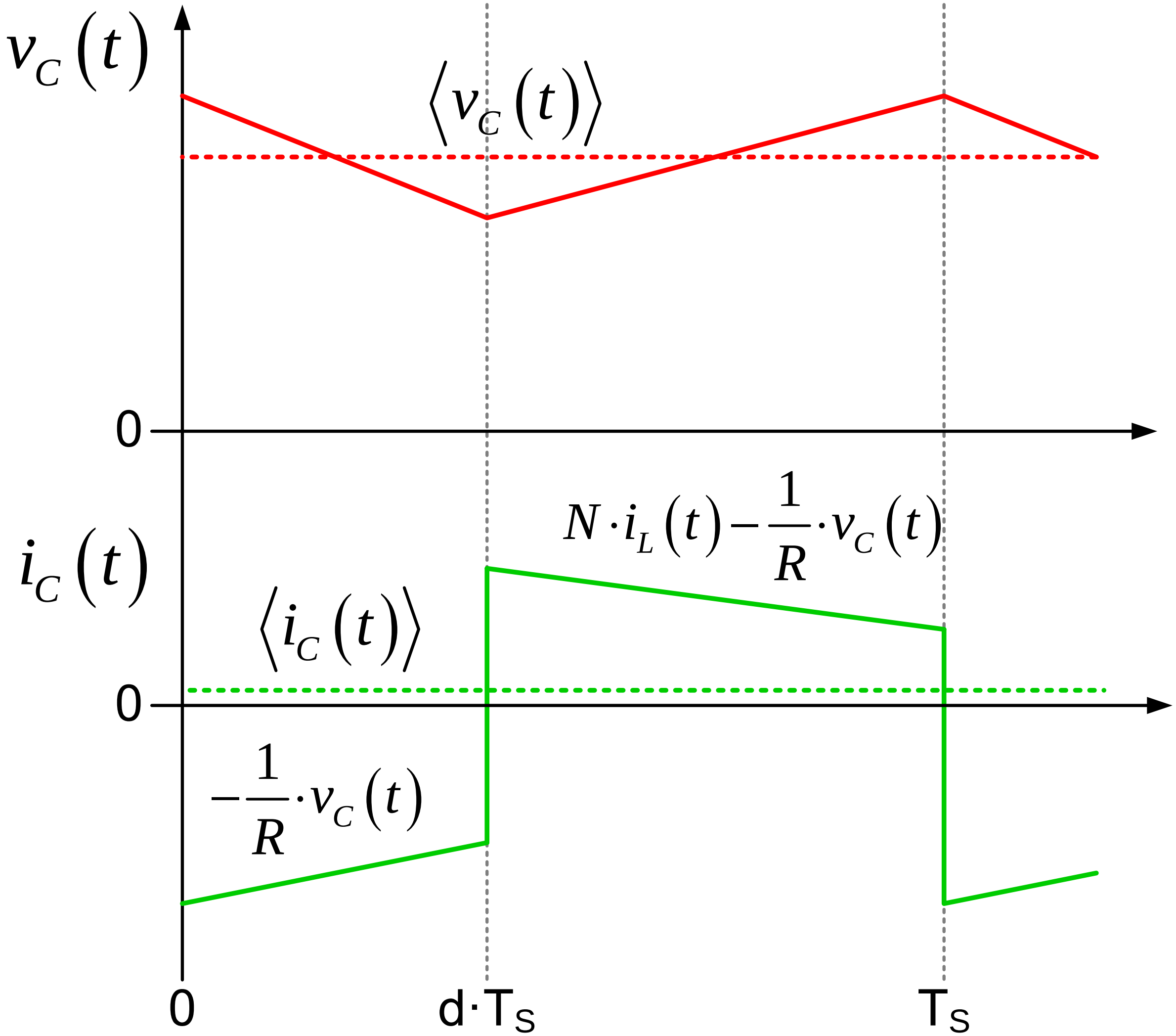
EMBEDDED POWER LABS

Sketching The Inductor Waveforms



EMBEDDED POWER LABS

Sketching The Capacitor Waveforms



Averaging The Circuit Equations

On Time

$$v_L(t) = v_g(t) - R_{ON} \cdot i_L(t)$$

$$i_C(t) = -\frac{1}{R} \cdot v_C(t)$$

$$i_g(t) = i_L(t)$$

Off Time

$$v_L(t) = -N \cdot \left(V_D + v_c(t)\right)$$

$$i_C(t) = N \cdot i_L(t) - \frac{1}{R} \cdot v_C(t)$$

$$i_g(t) = 0$$

$$v_L(t) = \left\langle v_g(t) \right\rangle - R_{ON} \cdot \left\langle i_L(t) \right\rangle$$

$$i_C(t) = -\frac{1}{R} \cdot \left\langle v_C(t) \right\rangle$$

$$i_g(t) = \left\langle i_L(t) \right\rangle$$

$$v_L(t) = -N \cdot \left(V_D + \left\langle v_c(t) \right\rangle\right)$$

$$i_C(t) = N \cdot \left\langle i_L(t) \right\rangle - \frac{1}{R} \cdot \left\langle v_C(t) \right\rangle$$

$$i_g(t) = 0$$

EMBEDDED POWER LABS

Averaging The Inductor Voltage

$$\langle v_L(t)\rangle = d(t)\cdot\left(\langle v_g(t)\rangle - R_{ON}\cdot\langle i_L(t)\rangle\right)$$
$$+d'(t)\cdot\left(-N\cdot\left(V_D+\langle v_c(t)\rangle\right)\right)$$
$$= d(t)\cdot\langle v_g(t)\rangle - d(t)\cdot R_{ON}\cdot\langle i_L(t)\rangle$$
$$-d'(t)\cdot N\cdot\langle v_c(t)\rangle - d'(t)\cdot N\cdot V_D$$

$$L\cdot\frac{d\langle i_L(t)\rangle}{dt} = \langle v_L(t)\rangle$$
$$= d(t)\cdot\langle v_g(t)\rangle - d(t)\cdot R_{ON}\cdot\langle i_L(t)\rangle$$
$$-d'(t)\cdot N\cdot\langle v_c(t)\rangle - d'(t)\cdot N\cdot V_D$$

EMBEDDED POWER LABS

Averaging The Capacitor Current

$$\left\langle i_C\left(t\right)\right\rangle = d\left(t\right)\cdot\left(-\frac{1}{R}\cdot\left\langle v_C\left(t\right)\right\rangle\right)$$

$$+d'\left(t\right)\cdot\left(N\cdot\left\langle i_L\left(t\right)\right\rangle-\frac{1}{R}\cdot\left\langle v_C\left(t\right)\right\rangle\right)$$

$$= d'\left(t\right)\cdot N\cdot\left\langle i_L\left(t\right)\right\rangle-\frac{1}{R}\cdot\left\langle v_C\left(t\right)\right\rangle$$

$$C\cdot\frac{d\left\langle v_c\left(t\right)\right\rangle}{dt} = \left\langle i_C\left(t\right)\right\rangle$$

$$= d'\left(t\right)\cdot N\cdot\left\langle i_L\left(t\right)\right\rangle-\frac{1}{R}\cdot\left\langle v_C\left(t\right)\right\rangle$$

EMBEDDED POWER LABS

Averaging The Input Current/ Perturbation

$$\left\langle i_g(t)\right\rangle = d(t)\cdot\left\langle i_L(t)\right\rangle + d'(t)\cdot 0$$
$$= d(t)\cdot\left\langle i_L(t)\right\rangle$$

Perturbing The Inputs

$$d(t) = D + \hat{d}(t)$$
$$d'(t) = 1 - d(t) = 1 - D - \hat{d}(t)$$
$$\left\langle V_g(t)\right\rangle = V_g + \hat{v}_g(t)$$

Perturbed Circuit Variables

$$\left\langle i_L(t)\right\rangle = I_L + \hat{i}_L(t)$$
$$\left\langle v_C(t)\right\rangle = V_C + \hat{v}_C(t)$$
$$\left\langle i_g(t)\right\rangle = I_g + \hat{i}_g(t)$$

Substitute Expressions Into Averaged Differential Equations

Averaging The Inductor Voltage

!

$$L \cdot \frac{d\left\langle i_L(t)\right\rangle}{dt} = \left\langle v_L(t)\right\rangle$$

$$= d(t)\cdot\left\langle v_g(t)\right\rangle - d(t)\cdot R_{ON}\cdot\left\langle i_L(t)\right\rangle$$

$$-d'(t)\cdot N\cdot\left\langle v_c(t)\right\rangle - d'(t)\cdot N\cdot V_D$$

$$L \cdot \frac{d\left(I_L + \hat{i}_L(t)\right)}{dt} = \left(D + \hat{d}(t)\right)\cdot\left(V_g + \hat{v}_g(t)\right) - \left(D + \hat{d}(t)\right)\cdot R_{ON}\cdot\left(I_L + \hat{i}_L(t)\right)$$

$$-\left(1 - D - \hat{d}(t)\right)\cdot N\cdot\left(V_C + \hat{v}_C(t)\right) - \left(1 - D - \hat{d}(t)\right)\cdot N\cdot V_D$$

Averaging The Inductor Voltage

!

$$L\cdot\frac{d\left(I_L\right)}{dt}+L\cdot\frac{d\left(\hat{i}_L\left(t\right)\right)}{dt}=D\cdot V_g+D\cdot\hat{v}_g\left(t\right)+V_g\cdot\hat{d}\left(t\right)+\hat{d}\left(t\right)\cdot\hat{v}_g\left(t\right)$$

$$-D\cdot R_{ON}\cdot I_L-D\cdot R_{ON}\cdot\hat{i}_L\left(t\right)-R_{ON}vI_L\cdot\hat{d}\left(t\right)-R_{ON}\cdot\hat{d}\left(t\right)\cdot\hat{i}_L\left(t\right)$$

$$-N\cdot V_C-N\cdot\hat{v}_C\left(t\right)+D\cdot N\cdot V_C+D\cdot N\cdot\hat{v}_C\left(t\right)+N\cdot V_C\cdot\hat{d}\left(t\right)+N\cdot\hat{v}_C\left(t\right)\cdot\hat{d}\left(t\right)$$

$$-N\cdot V_D+D\cdot N\cdot V_D+N\cdot V_D\cdot\hat{d}\left(t\right)$$

Averaging The Inductor Voltage

!

$$L\cdot\frac{d\left(I_L\right)}{dt}+L\cdot\frac{d\left(\hat{i}_L(t)\right)}{dt}=D\cdot V_g-D\cdot R_{ON}\cdot I_L-N\cdot V_C+D\cdot N\cdot V_C-N\cdot V_D+D\cdot N\cdot V_D$$

$$+D\cdot\hat{v}_g(t)$$

$$+V_g\cdot\hat{d}(t)-R_{ON}\cdot I_L\cdot\hat{d}(t)+N\cdot V_C\cdot\hat{d}(t)+N\cdot V_D\cdot\hat{d}(t)$$

$$-D\cdot R_{ON}\cdot\hat{i}_L(t)$$

$$-N\cdot\hat{v}_C(t)+D\cdot N\cdot\hat{v}_C(t)$$

$$+\hat{d}(t)\cdot\hat{v}_g(t)-R_{ON}\cdot\hat{d}(t)\cdot\hat{i}_L(t)+N\cdot\hat{v}_C(t)\cdot\hat{d}(t)$$

EMBEDDED POWER LABS

Averaging The Inductor Voltage

!

$$L \cdot \frac{d\left(I_L\right)}{dt} = 0$$

$$= D \cdot V_g - D \cdot R_{ON} \cdot I_L - N \cdot V_C + D \cdot N \cdot V_C - N \cdot V_D + D \cdot N \cdot V_D$$

$$= D \cdot \left(V_g - R_{ON} \cdot I_L\right) - \left(1 - D\right) \cdot N \cdot V_C - \left(1 - D\right) \cdot N \cdot V_D$$

$$D' \cdot N \cdot V_C = D \cdot \left(V_g - R_{ON} \cdot I_L\right) - D' \cdot N \cdot V_D$$

$$V_C = \frac{D}{D'} \cdot \frac{1}{N} \cdot \left(V_g - R_{ON} \cdot I_L\right) - V_D$$

Get The Same Result By Inductor Volt-Second Balance Calculation

EMBEDDED POWER LABS

Averaging The Inductor Voltage

$$L\frac{d\left\langle i_L(t)\right\rangle}{dt} = D\cdot\hat{v}_g(t)$$

$$+V_g\cdot\hat{d}(t) - R_{ON}\cdot I_L\cdot\hat{d}(t) + N\cdot V_C\cdot\hat{d}(t) + N\cdot V_D\cdot\hat{d}(t)$$

$$-D\cdot R_{ON}\cdot\hat{i}_L(t)$$

$$-N\cdot\hat{v}_C(t) + D\cdot N\cdot\hat{v}_C(t)$$

$$= D\cdot\hat{v}_g(t)$$

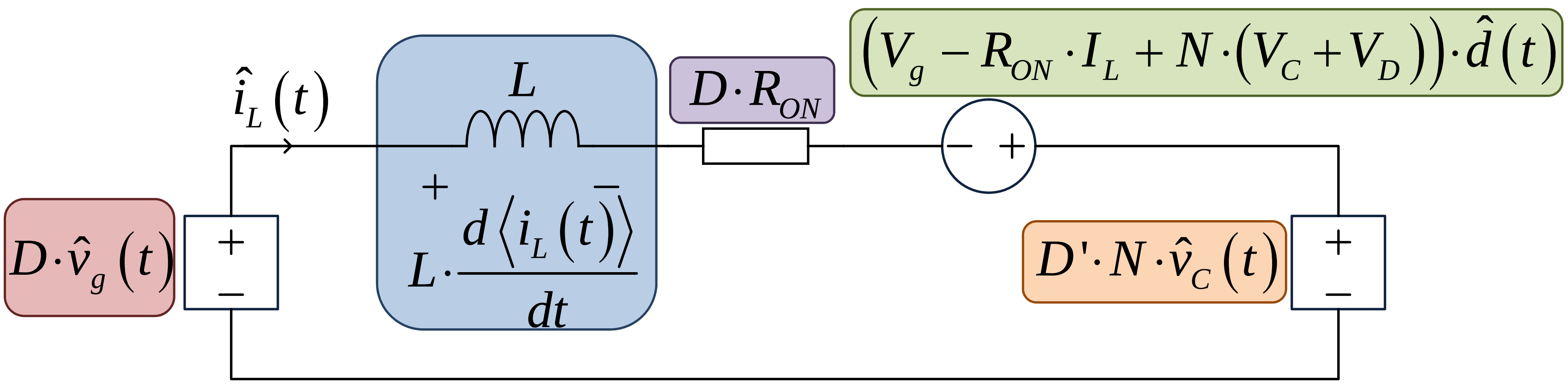
$$+\left(V_g - R_{ON}\cdot I_L + N\cdot\left(V_C + V_D\right)\right)\cdot\hat{d}(t)$$

$$-D\cdot R_{ON}\cdot\hat{i}_L(t)$$

$$-(1-D)\cdot N\cdot\hat{v}_C(t)$$

Construct The Model

$$L \cdot \frac{d\langle i_L(t)\rangle}{dt} = D \cdot \hat{v}_g(t) + \left(V_g - R_{ON} \cdot I_L + N \cdot \left(V_C + V_D\right)\right) \cdot \hat{d}(t) - D \cdot R_{ON} \cdot \hat{i}_L(t) - D' \cdot N \cdot \hat{v}_C(t)$$



Averaging The Capacitor Current

!

$$C\frac{d\langle v_c(t)\rangle}{dt} = d'(t)\cdot N\cdot\langle i_L(t)\rangle - \frac{1}{R}\cdot\langle v_C(t)\rangle$$

$$C\frac{d\left(V_C+\hat{v}_C(t)\right)}{dt} = \left(1-D-\hat{d}(t)\right)\cdot N\cdot\left(I_L+\hat{i}_L(t)\right) - \frac{1}{R}\cdot\left(V_C+\hat{v}_C(t)\right)$$

$$C\frac{d(V_C)}{dt} + C\frac{d\left(\hat{v}_C(t)\right)}{dt} = N\cdot I_L + N\cdot\hat{i}_L(t) - D\cdot N\cdot I_L - D\cdot N\cdot\hat{i}_L(t)$$

$$-N\cdot I_L\cdot\hat{d}(t) - N\cdot\hat{i}_L(t)\cdot\hat{d}(t)$$

$$-\frac{1}{R}\cdot V_C - \frac{1}{R}\cdot\hat{v}_C(t)$$

EMBEDDED POWER LABS

Averaging The Capacitor Current

!

$$C\frac{d\left(V_C\right)}{dt}+C\frac{d\left(\hat{v}_C\left(t\right)\right)}{dt}=N\cdot I_L-D\cdot N\cdot I_L-\frac{1}{R}\cdot V_C$$

$$+N\cdot\hat{i}_L\left(t\right)-D\cdot N\cdot\hat{i}_L\left(t\right)$$

$$-N\cdot I_L\cdot\hat{d}\left(t\right)$$

$$-\frac{1}{R}\cdot\hat{v}_C\left(t\right)$$

$$C\frac{d\left(\hat{v}_C\left(t\right)\right)}{dt}=D'\cdot N\cdot\hat{i}_L\left(t\right)-N\cdot I_L\cdot\hat{d}\left(t\right)-\frac{1}{R}\cdot\hat{v}_C\left(t\right)$$

EMBEDDED POWER LABS

Averaging The Capacitor Current

!

$$C\frac{d\left(V_C\right)}{dt} = D'\cdot N\cdot I_L - \frac{1}{R}\cdot V = 0$$

$$D'\cdot N\cdot I_L = \frac{1}{R}\cdot V_C$$

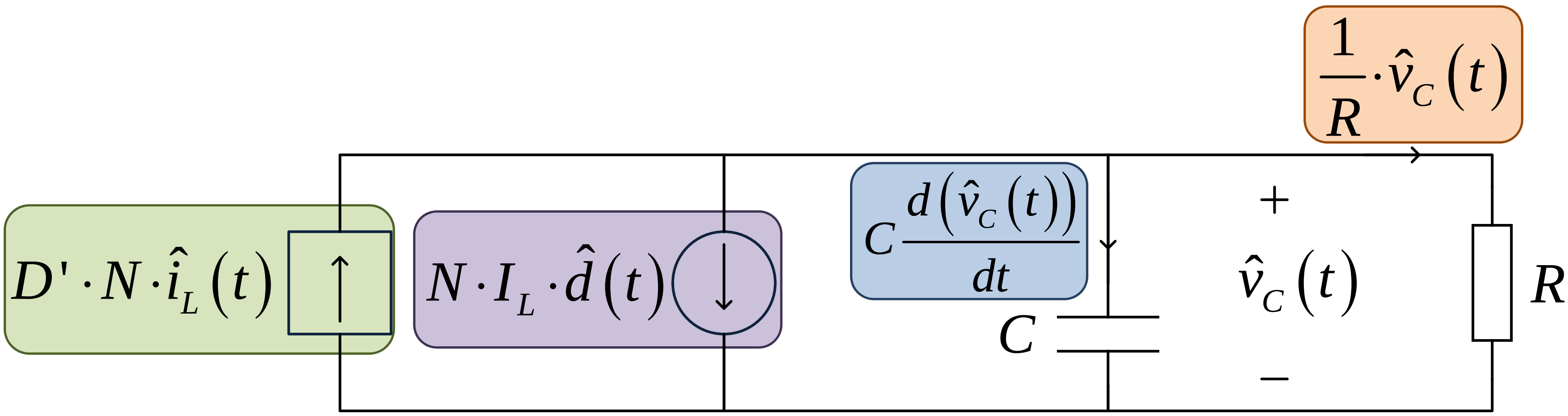
$$I_L = \frac{1}{D'}\cdot\frac{1}{N\cdot R}\cdot V_C$$

Get The Same Result By Capacitor Charge Balance Calculation

EMBEDDED POWER LABS

Constructing The Model

$$C\frac{d\left(\hat{v}_C(t)\right)}{dt} = D'\cdot N\cdot\hat{i}_L(t) - N\cdot I_L\cdot\hat{d}(t) - \frac{1}{R}\cdot\hat{v}_C(t)$$



Averaging The Input Current

$$\langle i_g(t)\rangle = d(t)\cdot\langle i_L(t)\rangle$$

$$I_g + \hat{i}_g(t) = \left(D + \hat{d}(t)\right)\cdot\left(I_L + \hat{i}_L(t)\right)$$

$$= D\cdot I_L + D\cdot\hat{i}_L(t) + I_L\cdot\hat{d}(t) + \cancel{\hat{d}(t)\cdot\hat{i}_L(t)}$$

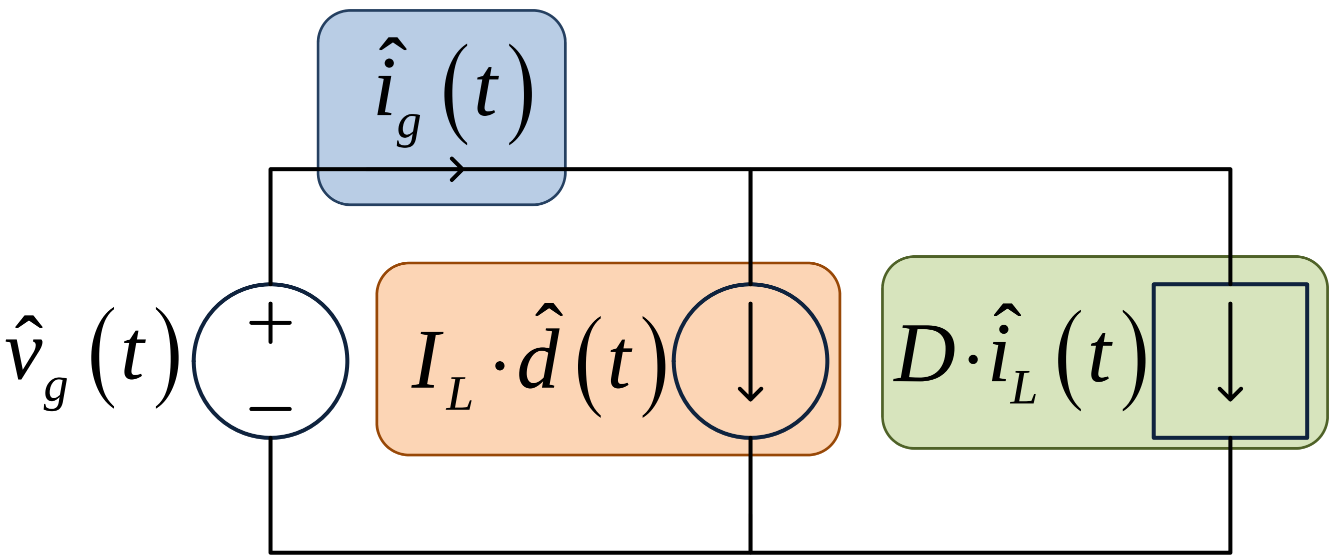
$$I_g = D\cdot I_L$$

$$\hat{i}_g(t) = D\cdot\hat{i}_L(t) + I_L\cdot\hat{d}(t)$$

EMBEDDED POWER LABS

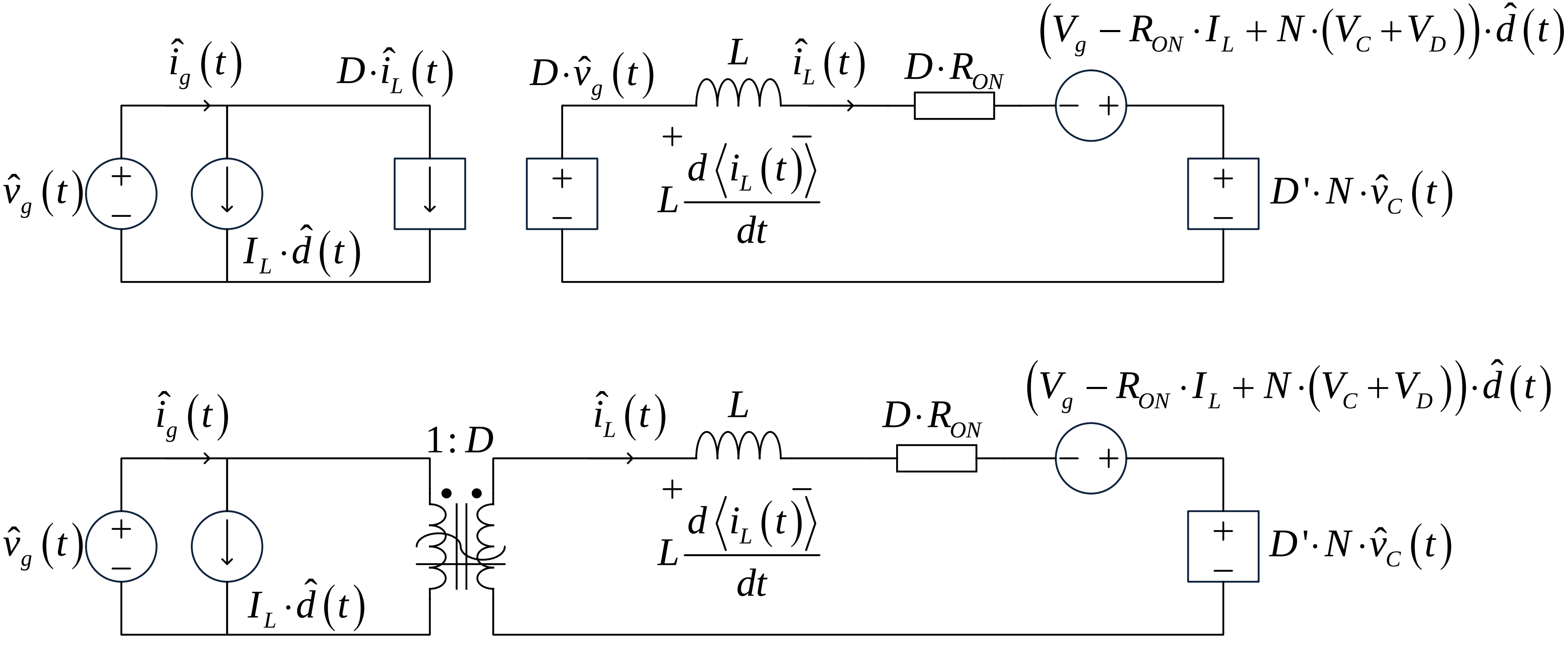
Constructing The Model

$$\hat{i}_g(t) = D \cdot \hat{i}_L(t) + I_L \cdot \hat{d}(t)$$

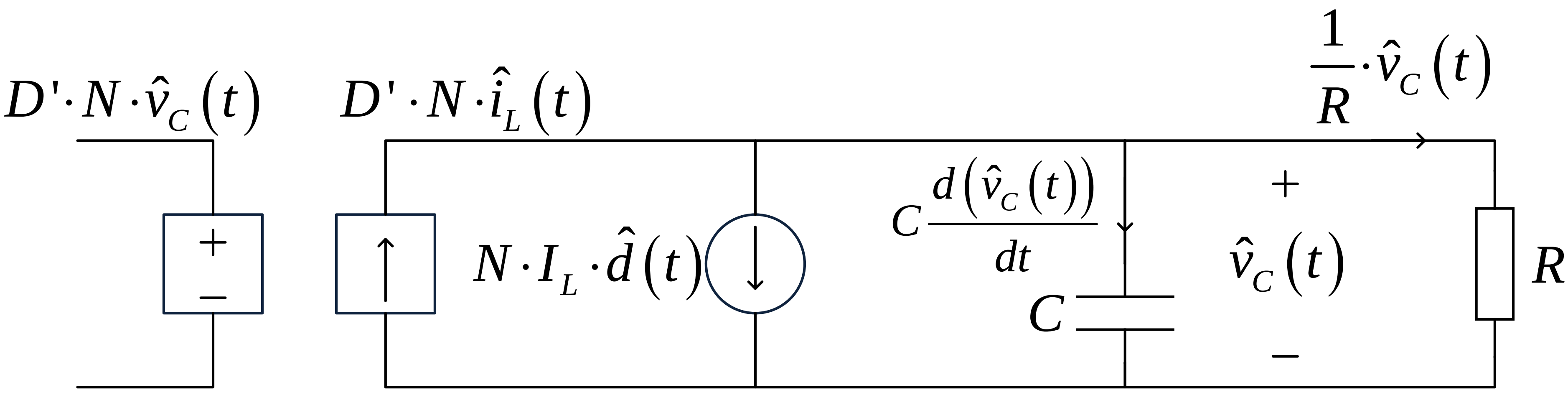


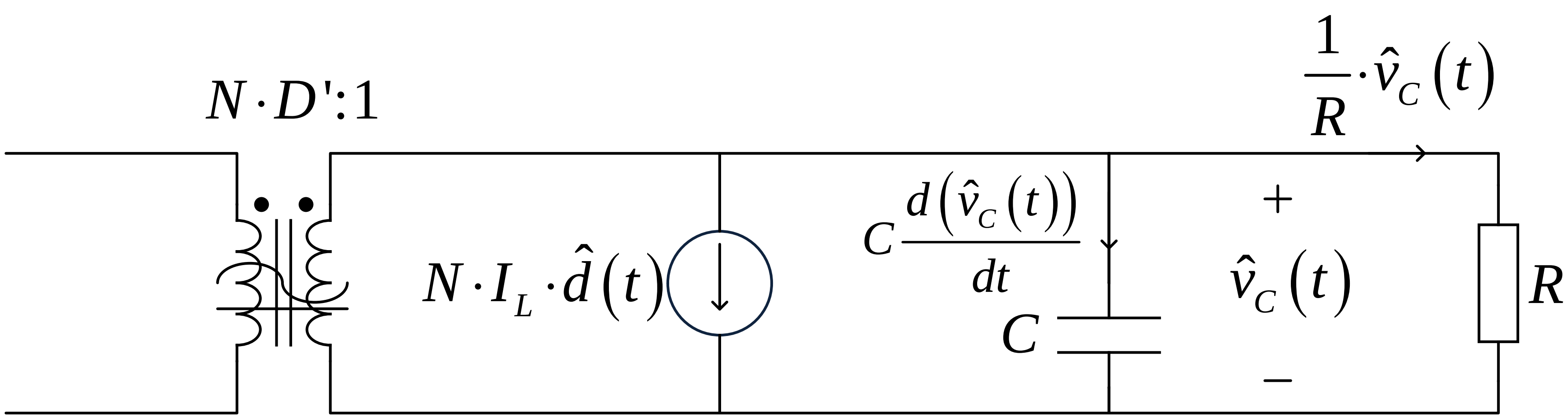
EMBEDDED POWER LABS

Completing The Flyback Model



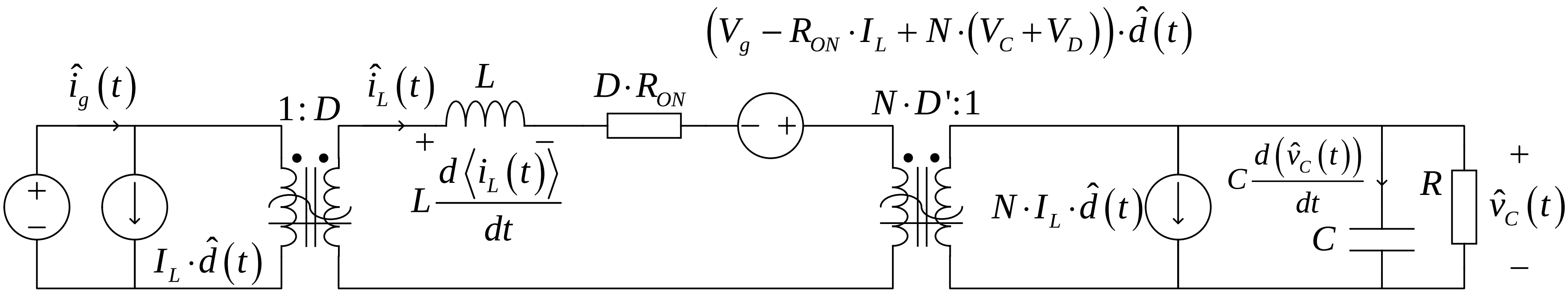
Completing The Flyback Model





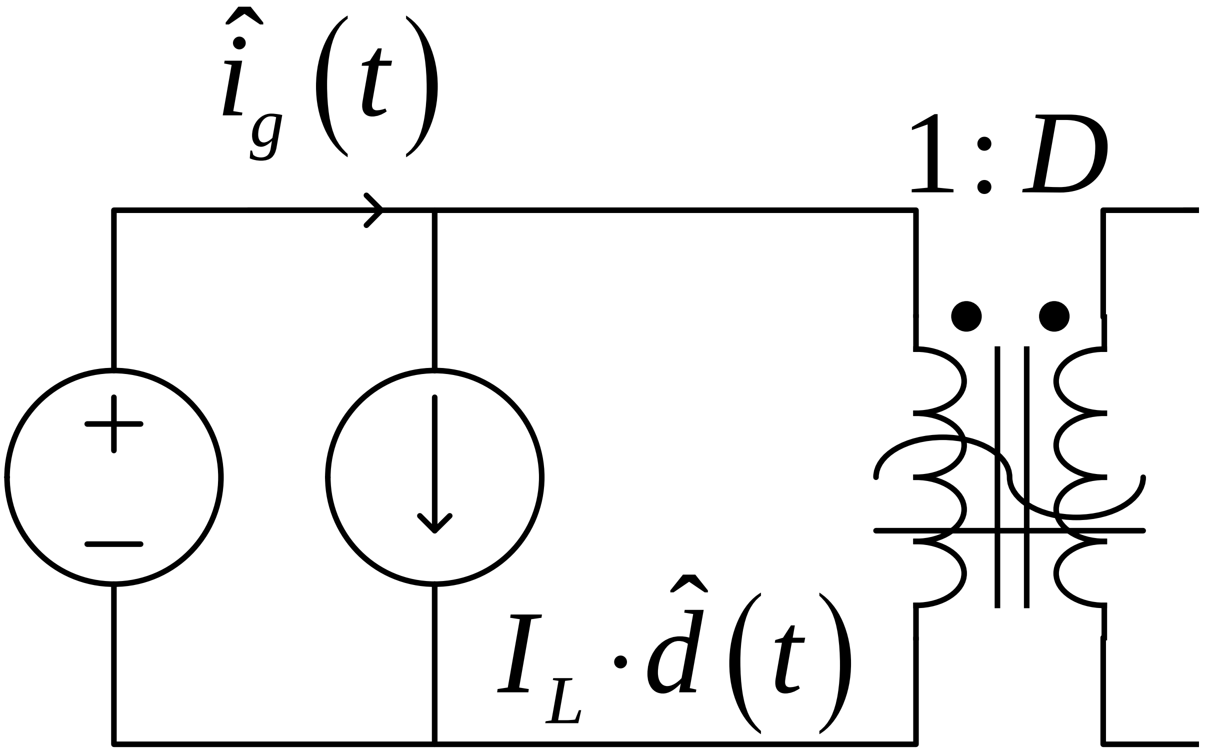
EMBEDDED POWER LABS

Complete Flyback Model

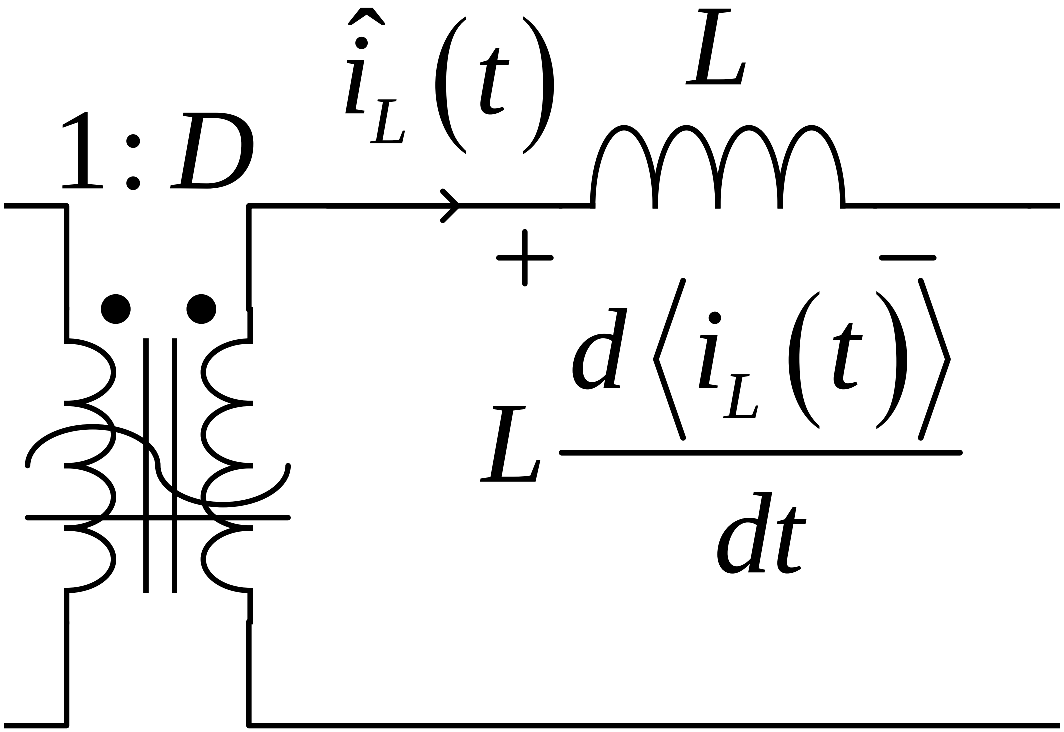


EMBEDDED POWER LABS

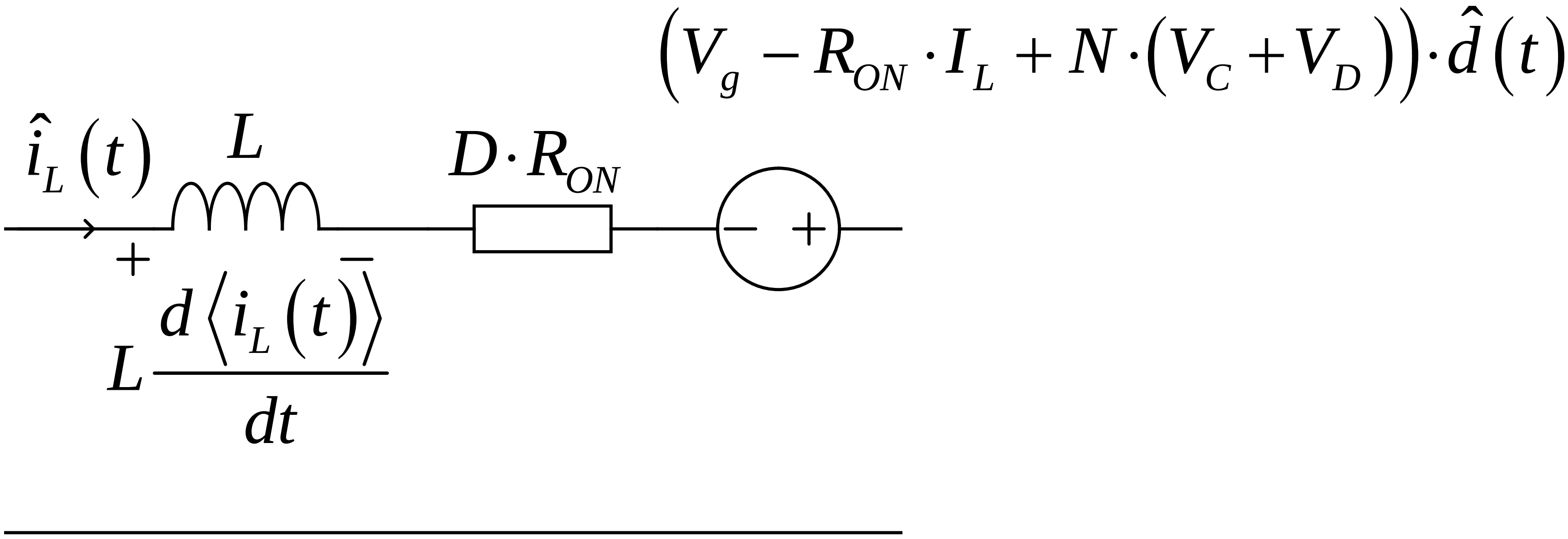
Complete Flyback Model



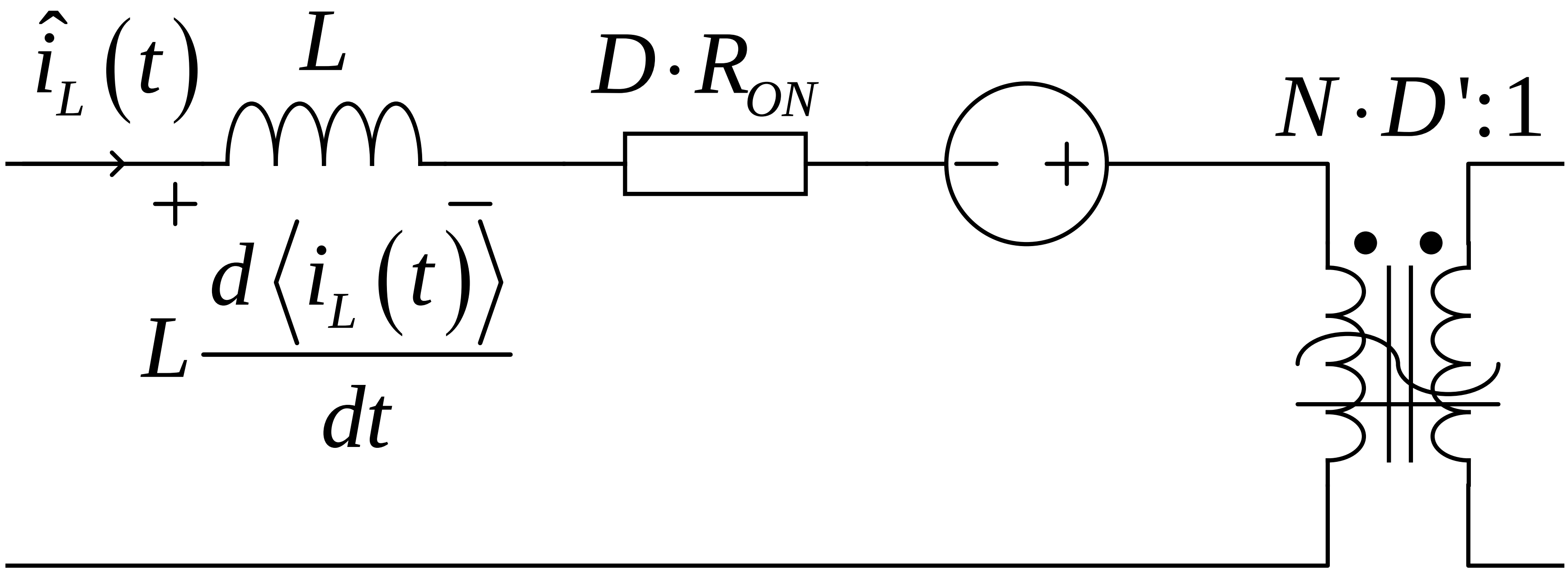
Complete Flyback Model



Complete Flyback Model

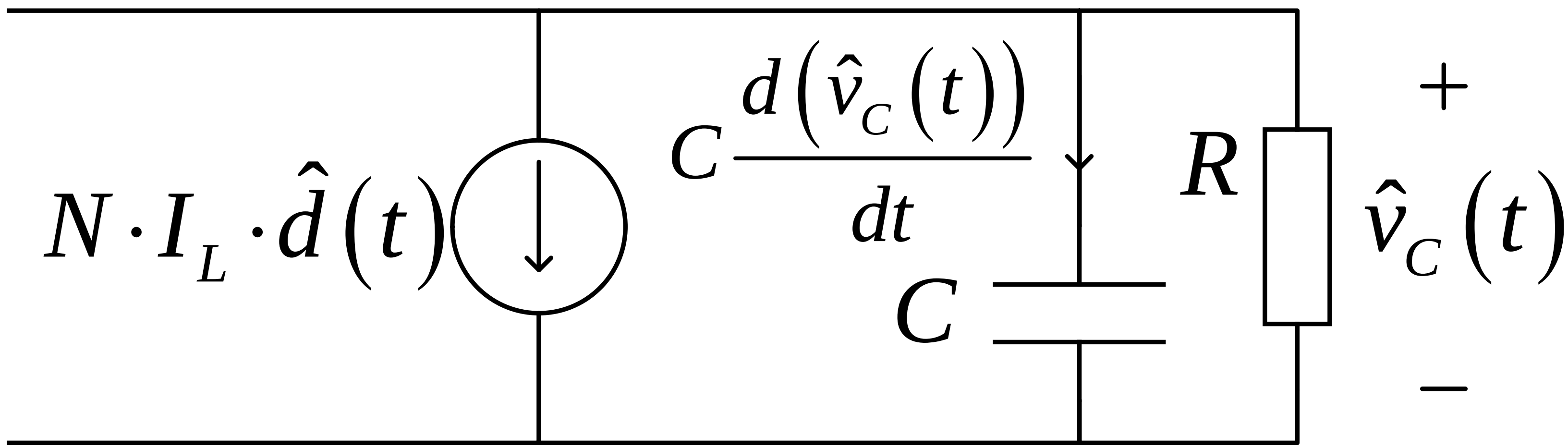


Complete Flyback Model



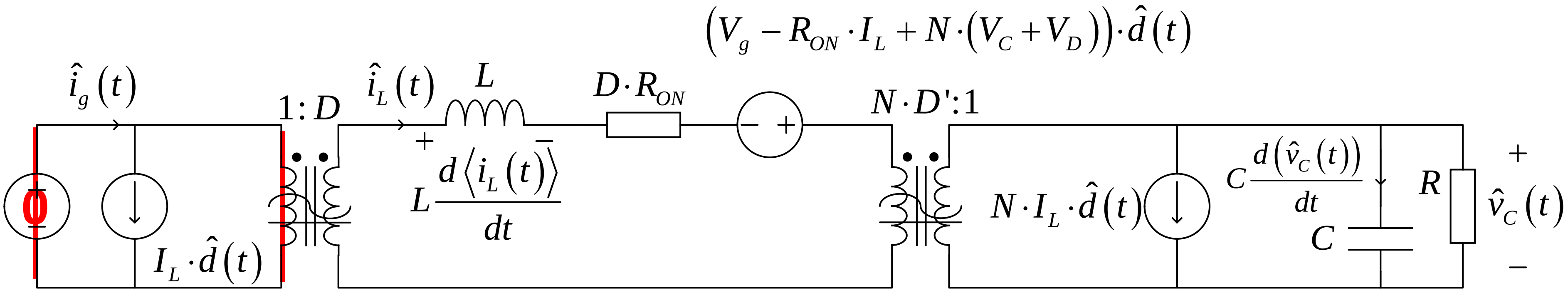
EMBEDDED POWER LABS

Complete Flyback Model

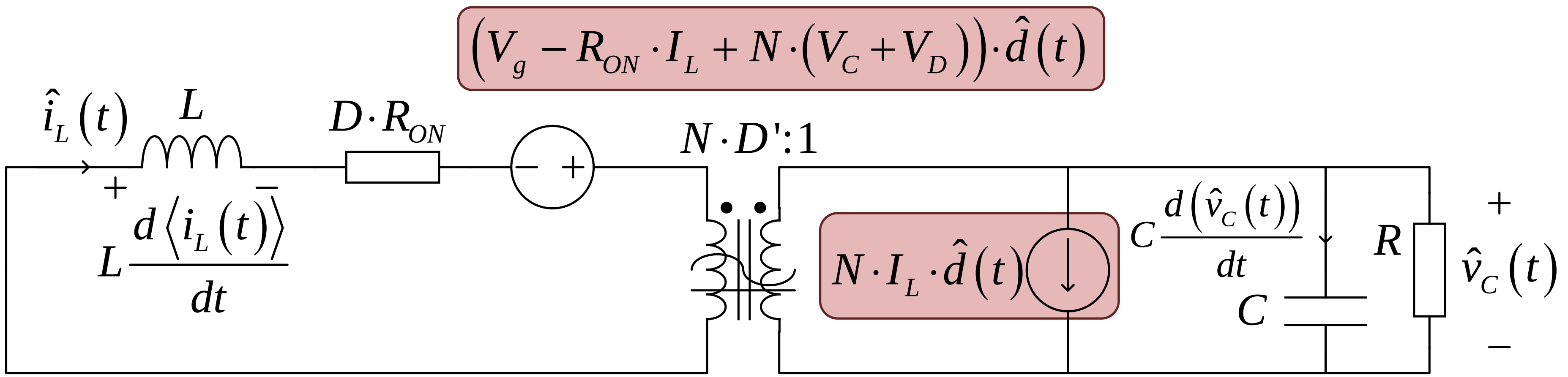


Control-To-Output Transfer Function

$$G_{vd}(s) = \left.\frac{\hat{v}_C(s)}{\hat{d}(s)}\right|_{\hat{v}_g=0}$$



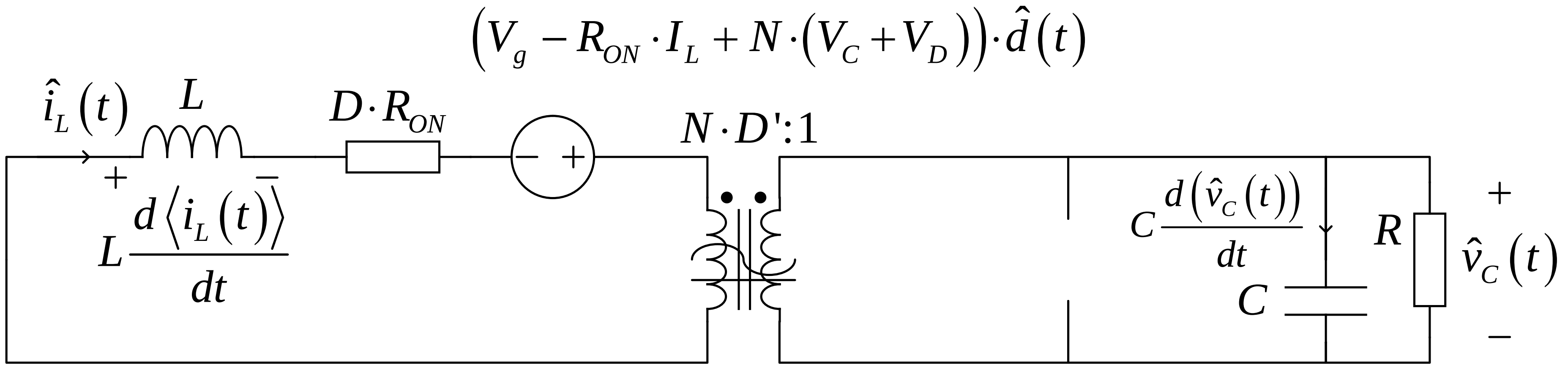
Control-To-Output Transfer Function

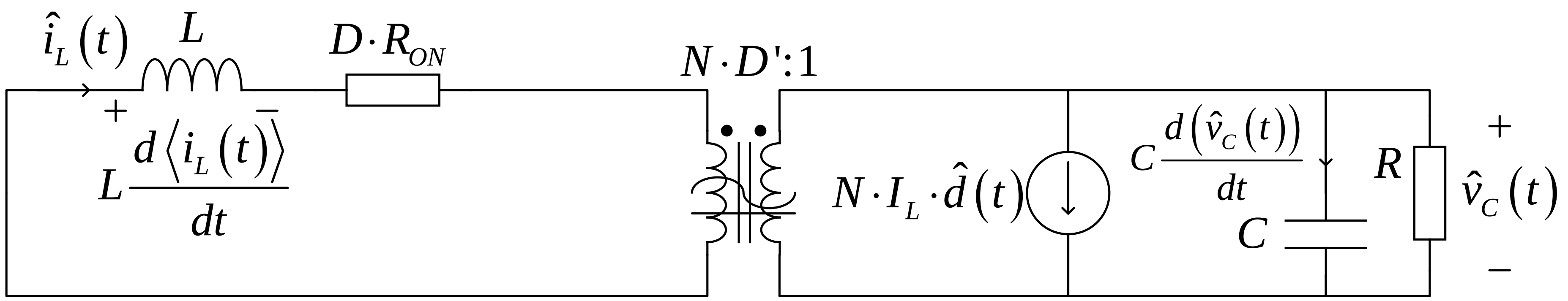


Still Have Two dhat Terms

Solve By Using Superposition

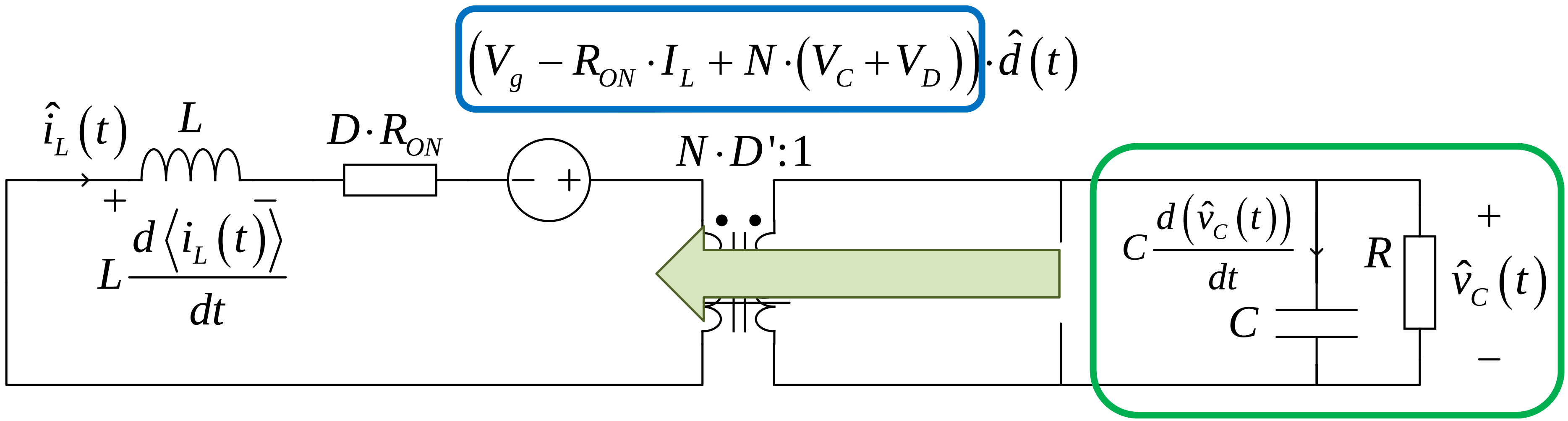
Control-To-Output Transfer Function

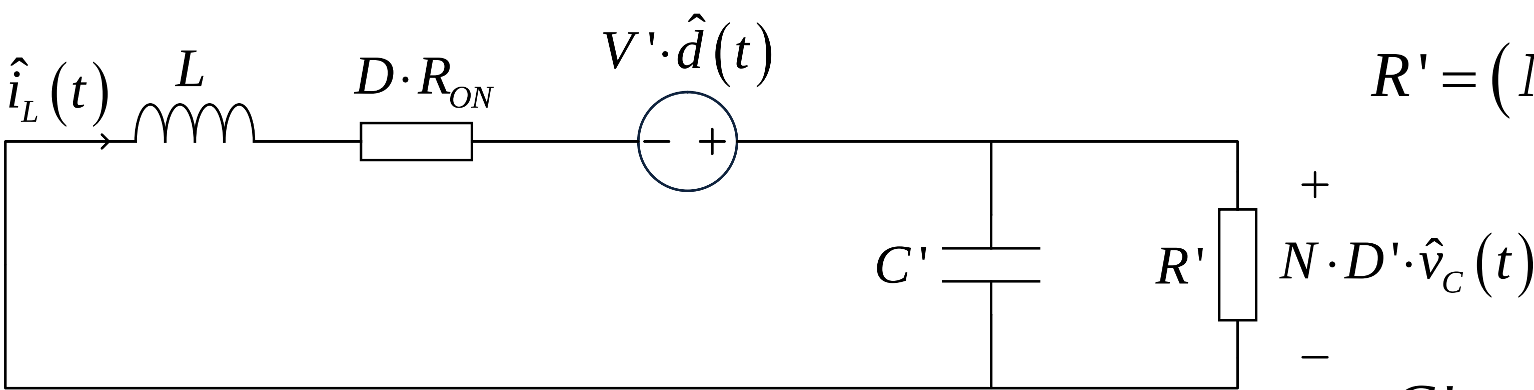




Control-To-Output Transfer Function

$$V' = V_g - R_{ON} \cdot I_L + N \cdot (V_C + V_D)$$





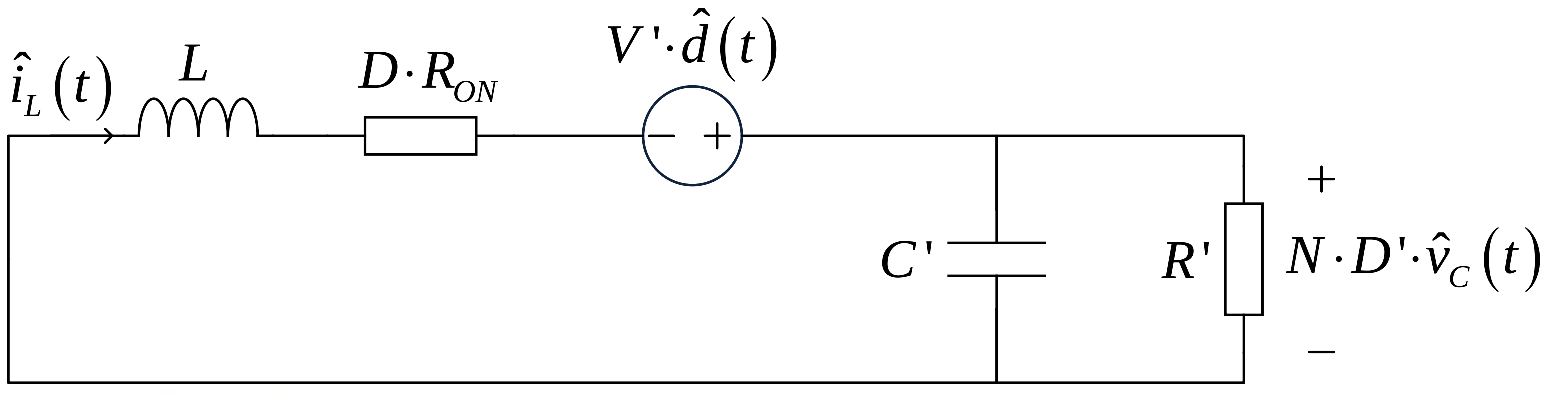
$$R' = (N \cdot D')^2 \cdot R$$

$$C' = \frac{C}{(N \cdot D')^2}$$

Control-To-Output Transfer Function

$$N \cdot D' \cdot \hat{v}_C(s) = \frac{R' \| \frac{1}{s \cdot C'}}{s \cdot L + D \cdot R_{ON} + R' \| \frac{1}{s \cdot C'}} \cdot V' \cdot \hat{d}(t)$$

$$G_{vd1}(s) = \frac{\hat{v}_C(s)}{\hat{d}(t)} = \frac{V'}{N \cdot D'} \cdot \frac{R' \| \frac{1}{s \cdot C'}}{s \cdot L + D \cdot R_{ON} + R' \| \frac{1}{s \cdot C'}}$$



EMBEDDED POWER LABS

Control-To-Output Transfer Function

$$R'\|\frac{1}{s\cdot C'}=\frac{R'\cdot\frac{1}{s\cdot C'}}{R'+\frac{1}{s\cdot C'}}\cdot\frac{s\cdot C'}{s\cdot C'}=\frac{R'}{1+s\cdot R'\cdot C'}$$

$$\frac{R'\|\frac{1}{s\cdot C'}}{s\cdot L+D\cdot R_{ON}+R'\|\frac{1}{s\cdot C'}}=\frac{\frac{R'}{1+s\cdot R'\cdot C'}}{s\cdot L+D\cdot R_{ON}+\frac{R'}{1+s\cdot R'\cdot C'}}\cdot\frac{1+s\cdot R'\cdot C'}{1+s\cdot R'\cdot C'}$$

$$=\frac{R'}{s\cdot L+s^2\cdot R'\cdot C'\cdot L+D\cdot R_{ON}+s\cdot D\cdot R_{ON}\cdot R'\cdot C'+R'}$$

$$=\frac{R'}{R'+D\cdot R_{ON}+s\cdot L+s\cdot D\cdot R_{ON}\cdot R'\cdot C'+s^2\cdot R'\cdot C'\cdot L}\cdot\frac{\frac{1}{R'}}{\frac{1}{R'}}$$

$$=\frac{1}{1+\frac{D\cdot R_{ON}}{R'}+s\cdot\left(\frac{L}{R'}+D\cdot R_{ON}\cdot C'\right)+s^2\cdot C'\cdot L}$$

Control-To-Output Transfer Function

$$\frac{R' \| \frac{1}{s\cdot C'}}{s\cdot L + D\cdot R_{ON} + R' \| \frac{1}{s\cdot C'}} = \frac{1}{1 + \frac{D\cdot R_{ON}}{R'} + s\cdot\left(\frac{L}{R'} + D\cdot R_{ON}\cdot C'\right) + s^2\cdot C'\cdot L}$$

$$R' = (N\cdot D')^2\cdot R \qquad C' = \frac{C}{(N\cdot D')^2}$$

$$= \frac{1}{1 + \frac{D\cdot R_{ON}}{(N\cdot D')^2\cdot R} + s\cdot\left(\frac{L}{(N\cdot D')^2\cdot R} + D\cdot R_{ON}\cdot\frac{C}{(N\cdot D')^2}\right) + s^2\cdot\frac{C}{(N\cdot D')^2}\cdot L}$$

EMBEDDED POWER LABS

Control-To-Output Transfer Function

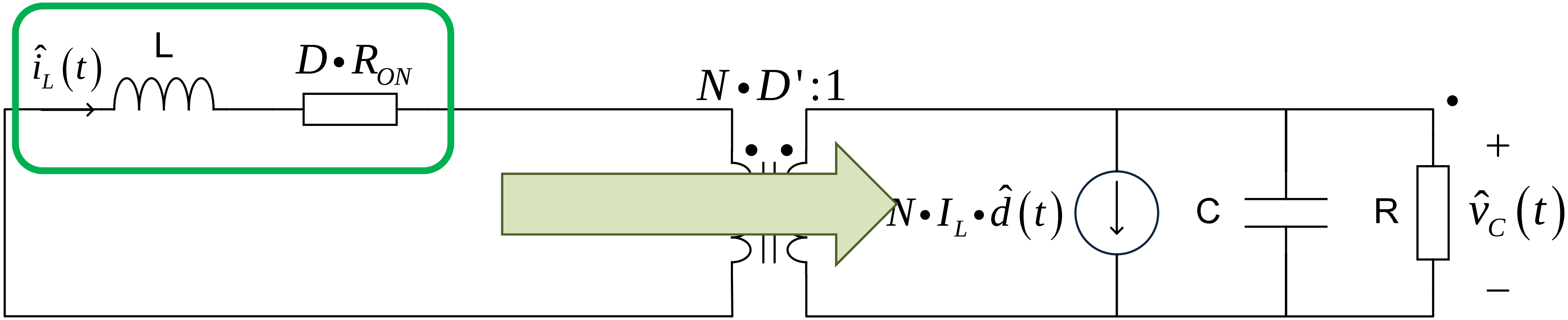
$$G_{vd1}(s) =$$

$$\frac{V'}{N \cdot D'} \cdot \frac{1}{1 + \frac{D \cdot R_{ON}}{(N \cdot D')^2 \cdot R} + s \cdot \left(\frac{L}{(N \cdot D')^2 \cdot R} + \frac{D \cdot R_{ON} \cdot C}{(N \cdot D')^2} \right) + s^2 \cdot \frac{L \cdot C}{(N \cdot D')^2}}$$

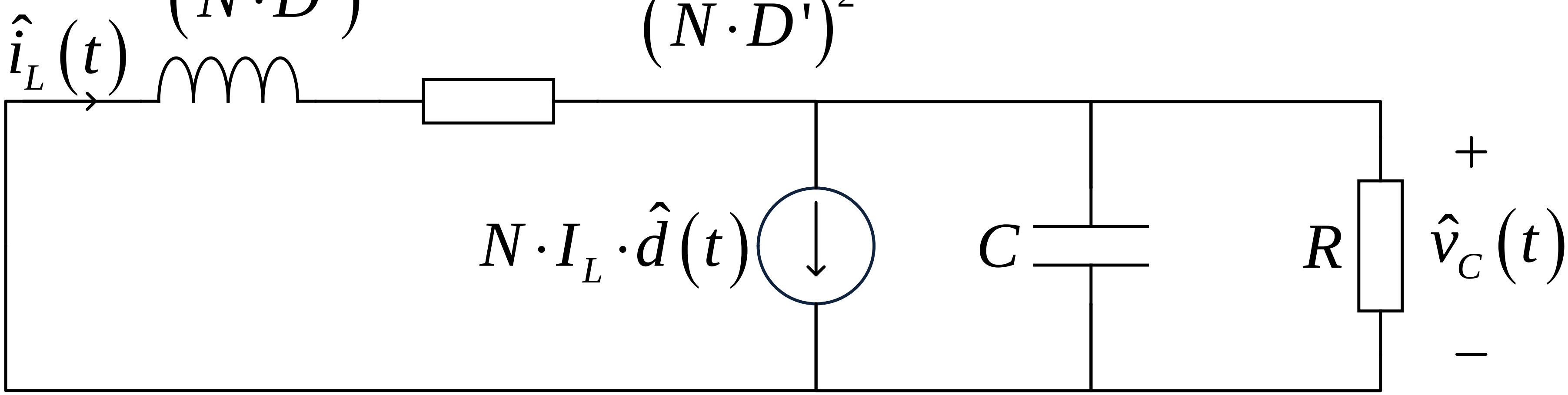
$$V' = V_g - R_{ON} \cdot I_L + N \cdot (V_C + V_D)$$

EMBEDDED POWER LABS

Control-To-Output Transfer Function



$$L' = \frac{L}{(N \cdot D')^2} \qquad R' = \frac{D \cdot R_{ON}}{(N \cdot D')^2}$$

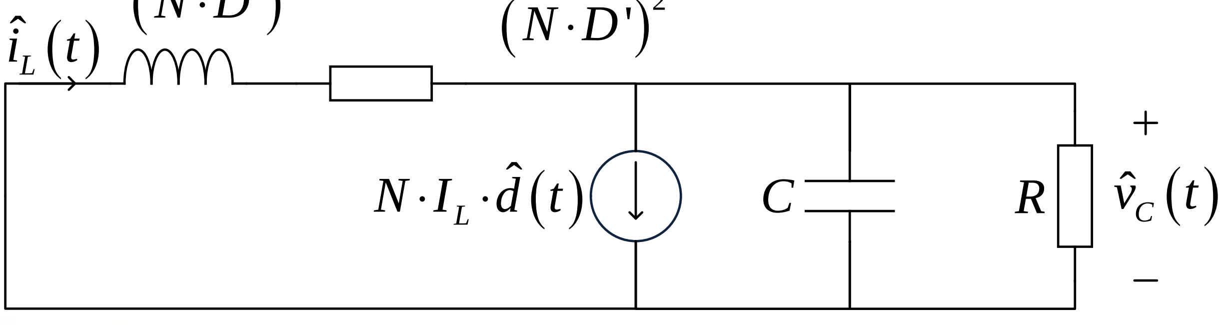


Control-To-Output Transfer Function

$$\hat{v}_C(t) = -N \cdot I_L \cdot \hat{d}(t) \cdot \left[(s \cdot L' + R') \parallel \left(\frac{1}{s \cdot C} \right) \parallel R \right]$$

$$= -N \cdot I_L \cdot \hat{d}(t) \cdot \frac{1}{\frac{1}{s \cdot L' + R'} + \frac{1}{\frac{1}{s \cdot C}} + \frac{1}{R}}$$

$$L' = \frac{L}{(N \cdot D')^2} \qquad R' = \frac{D \cdot R_{ON}}{(N \cdot D')^2}$$



Control-To-Output Transfer Function

$$\frac{1}{\frac{1}{s\cdot L'+R'}+\frac{1}{\frac{1}{s\cdot C}}+\frac{1}{R}}=\frac{1}{\frac{1}{s\cdot L'+R'}+s\cdot C+\frac{1}{R}}$$

$$=\frac{1}{\frac{1}{s\cdot L'+R'}+\frac{1+s\cdot R\cdot C}{R}}$$

$$=\frac{1}{\frac{R+\left(s\cdot L'+R'\right)\cdot\left(1+s\cdot R\cdot C\right)}{R\cdot\left(s\cdot L'+R'\right)}}$$

$$=\frac{R\cdot\left(s\cdot L'+R'\right)}{R+s\cdot L'+R'+s\cdot R\cdot C\cdot R'+s^2\cdot R\cdot L'\cdot C}$$

EMBEDDED POWER LABS

Control-To-Output Transfer Function

$$\frac{1}{\dfrac{1}{s\cdot L'+R'}+\dfrac{1}{\dfrac{1}{s\cdot C}}+\dfrac{1}{R}}=\frac{R\cdot\left(s\cdot L'+R'\right)}{R+R'+s\cdot L'+s\cdot R\cdot C\cdot R'+s^2\cdot R\cdot L'\cdot C}\cdot\frac{\dfrac{1}{R}}{\dfrac{1}{R}}$$

$$=\frac{s\cdot L'+R'}{1+\dfrac{R'}{R}+s\cdot\left(\dfrac{L'}{R}+R'\cdot C\right)+s^2\cdot L'\cdot C}$$

$$L'=\frac{L}{\left(N\cdot D'\right)^2}\qquad R'=\frac{D\cdot R_{ON}}{\left(N\cdot D'\right)^2}$$

EMBEDDED POWER LABS

Control-To-Output Transfer Function

$$= \frac{s\cdot\frac{L}{(N\cdot D')^2}+\frac{D\cdot R_{ON}}{(N\cdot D')^2}}{1+\frac{\frac{D\cdot R_{ON}}{(N\cdot D')^2}}{R}+s\cdot\left(\frac{\frac{L}{(N\cdot D')^2}}{R}+\frac{D\cdot R_{ON}}{(N\cdot D')^2}\cdot C\right)+s^2\cdot\frac{L}{(N\cdot D')^2}\cdot C}$$

$$= \frac{\frac{1}{(N\cdot D')^2}\cdot\left(s\cdot L+D\cdot R_{ON}\right)}{1+\frac{D\cdot R_{ON}}{(N\cdot D')^2\cdot R}+s\cdot\left(\frac{L}{(N\cdot D')^2\cdot R}+\frac{D\cdot R_{ON}\cdot C}{(N\cdot D')^2}\right)+s^2\cdot\frac{L\cdot C}{(N\cdot D')^2}}$$

EMBEDDED POWER LABS

Control-To-Output Transfer Function

$$G_{vd2}(s) = \frac{\hat{v}_C(t)}{\hat{d}(t)}$$

$$= -N \cdot I_L \cdot \frac{\frac{1}{(N \cdot D')^2} \cdot (s \cdot L + D \cdot R_{ON})}{1 + \frac{D \cdot R_{ON}}{(N \cdot D')^2 \cdot R} + s \cdot \left(\frac{L}{(N \cdot D')^2 \cdot R} + \frac{D \cdot R_{ON} \cdot C}{(N \cdot D')^2} \right) + s^2 \cdot \frac{L \cdot C}{(N \cdot D')^2}}$$

Control-To-Output Transfer Function

$$G_{vd}(s) = G_{vd1}(s) + G_{vd2}(s)$$

$$G_{vd}(s) = \frac{num\left(G_{vd}(s)\right)}{den\left(G_{vd}(s)\right)} = \frac{num\left(G_{vd1}(s)\right)}{den\left(G_{vd1}(s)\right)} + \frac{num\left(G_{vd2}(s)\right)}{den\left(G_{vd2}(s)\right)}$$

$$den\left(G_{vd1}(s)\right) = den\left(G_{vd2}(s)\right) = den\left(G_{vd}(s)\right)$$

$$G_{vd}(s) = \frac{num\left(G_{vd}(s)\right)}{den\left(G_{vd}(s)\right)} = \frac{\boxed{num\left(G_{vd1}(s)\right) + num\left(G_{vd2}(s)\right)}}{den\left(G_{vd}(s)\right)}$$

Control-To-Output Transfer Function

$$num\left(G_{vd1}(s)\right) = \frac{V'}{N \cdot D'}$$

$$num\left(G_{vd2}(s)\right) = -N \cdot I_L \cdot \frac{1}{\left(N \cdot D'\right)^2} \cdot \left(s \cdot L + D \cdot R_{ON}\right)$$

$$num\left(G_{vd1}(s)\right) + num\left(G_{vd2}(s)\right) = \frac{V'}{N \cdot D'} - N \cdot I_L \cdot \frac{1}{\left(N \cdot D'\right)^2} \cdot \left(s \cdot L + D \cdot R_{ON}\right)$$

EMBEDDED POWER LABS

Control-To-Output Transfer Function

!

$$num\left(G_{vd1}(s)\right)+num\left(G_{vd2}(s)\right)=\frac{V'\cdot N\cdot D'}{\left(N\cdot D'\right)^2}-\frac{N\cdot I_L\cdot\left(s\cdot L+D\cdot R_{ON}\right)}{\left(N\cdot D'\right)^2}$$

$$=\frac{V'\cdot N\cdot D'-N\cdot D\cdot R_{ON}\cdot I_L-s\cdot N\cdot I_L\cdot L}{\left(N\cdot D'\right)^2}$$

$$=\frac{V'\cdot N\cdot D'-N\cdot D\cdot R_{ON}\cdot I_L}{\left(N\cdot D'\right)^2}\cdot\left(1-s\cdot\frac{N\cdot I_L\cdot L}{V'\cdot N\cdot D'-N\cdot D\cdot R_{ON}\cdot I_L}\right)$$

$$=\frac{V'\cdot D'-D\cdot R_{ON}\cdot I_L}{N\cdot D'^2}\cdot\left(1-s\cdot\frac{N\cdot I_L\cdot L}{V'\cdot N\cdot D'-N\cdot D\cdot R_{ON}\cdot I_L}\right)$$

EMBEDDED POWER LABS

Control To Output Transfer Function

!

Quick check on the algebra: Are the units correct?

$$1 - s \cdot \frac{N \cdot I_L \cdot L}{V' \cdot N \cdot D' - N \cdot D \cdot R_{ON} \cdot I_L}$$

$$\Rightarrow \frac{\frac{1}{s} \cdot A \cdot \frac{v \cdot s}{A}}{v - \Omega \cdot A} \Rightarrow \frac{v}{v} \Rightarrow \text{Dimensionless}$$

EMBEDDED POWER LABS

Control-To-Output Transfer Function !

$$G_{vd}(s)=\frac{\dfrac{V'\cdot D'-D\cdot R_{ON}\cdot I_L}{N\cdot D'^2}\cdot\left(1-s\cdot\dfrac{N\cdot I_L\cdot L}{V'\cdot N\cdot D'-N\cdot D\cdot R_{ON}\cdot I_L}\right)}{1+\dfrac{D\cdot R_{ON}}{(N\cdot D')^2\cdot R}+s\cdot\left(\dfrac{L}{(N\cdot D')^2\cdot R}+\dfrac{D\cdot R_{ON}\cdot C}{(N\cdot D')^2}\right)+s^2\cdot\dfrac{L\cdot C}{(N\cdot D')^2}}$$

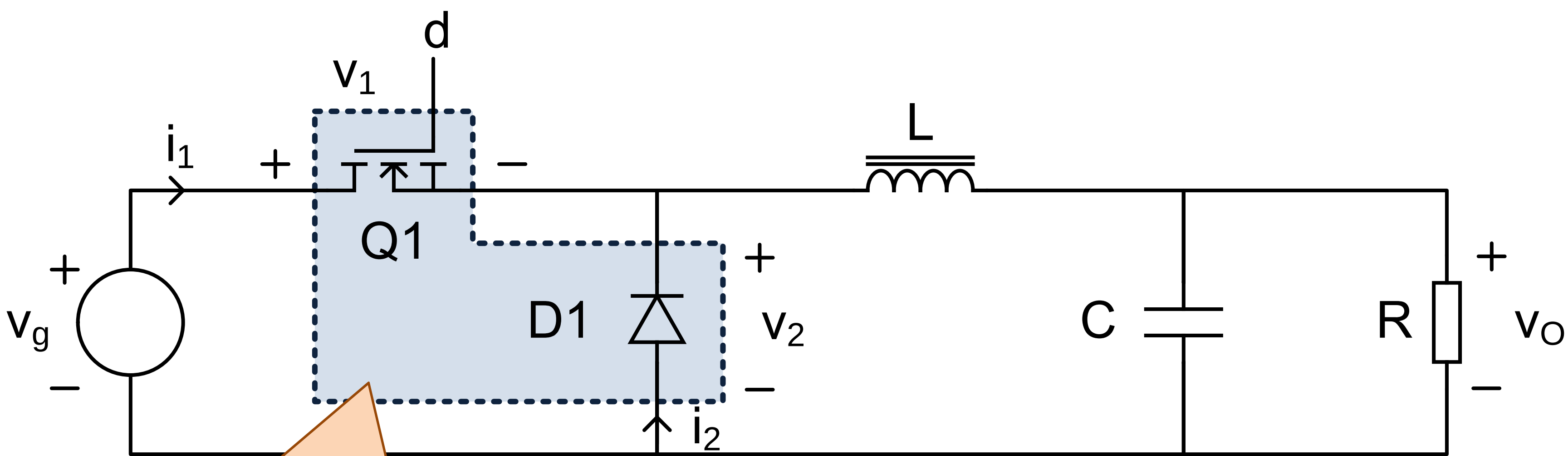
$$V'=V_g-R_{ON}\cdot I_L+N\cdot\left(V_C+V_D\right)$$

EMBEDDED POWER LABS

Averaging Summary

- Write Circuit Differential Equations For Each State In Terms Of Averaged Values
- Average Over One Switching Cycle
  – Inductor Current
  – Capacitor Voltages
- Perturb And Linearize
  – DC Terms Are Zero
  – Discard 2nd And Higher Order Terms
- Construct The Circuit Model

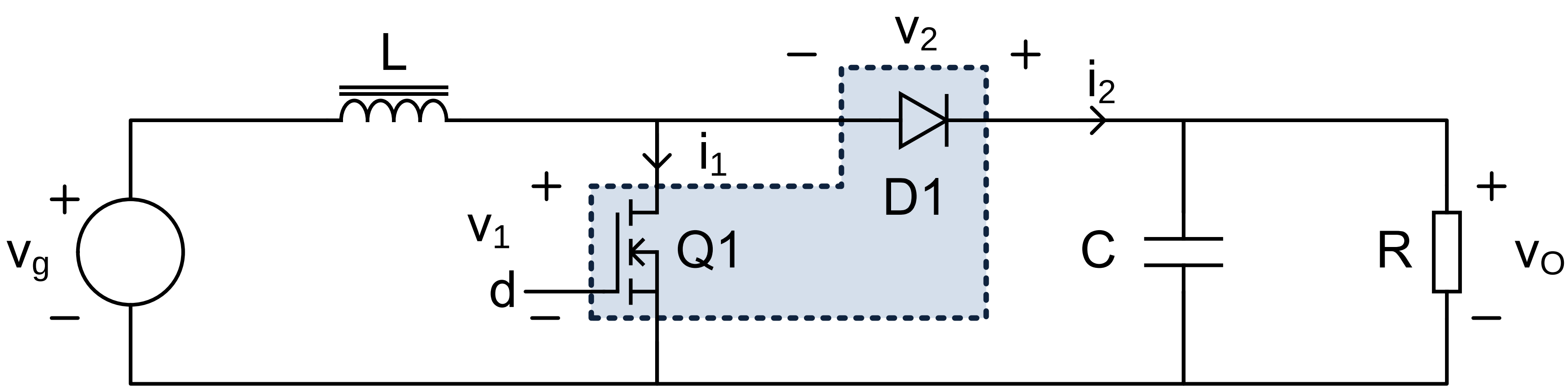
Averaged Switch Modeling

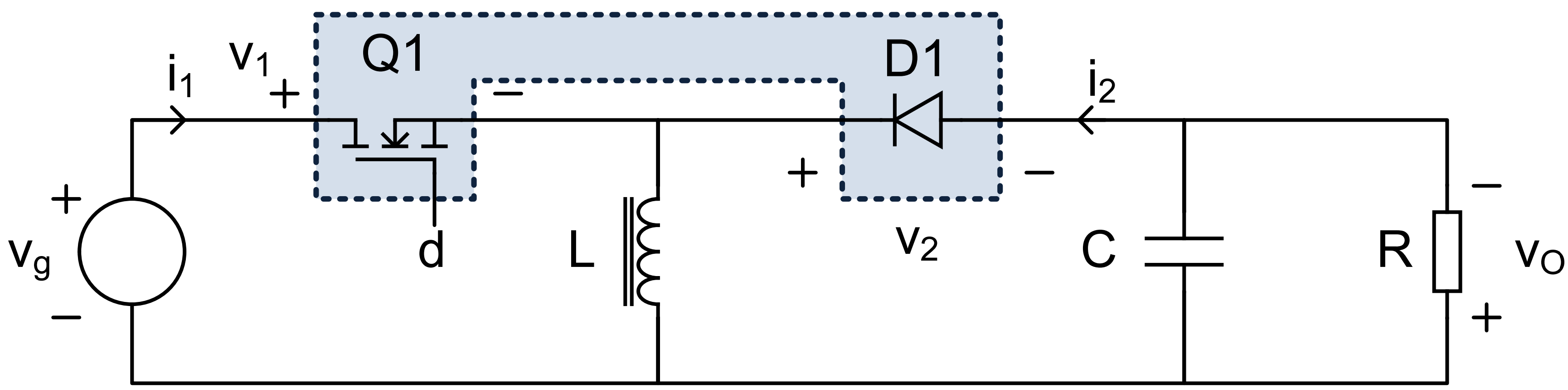


Two Port Switch Network Includes All Nonlinear And Time Varying Elements

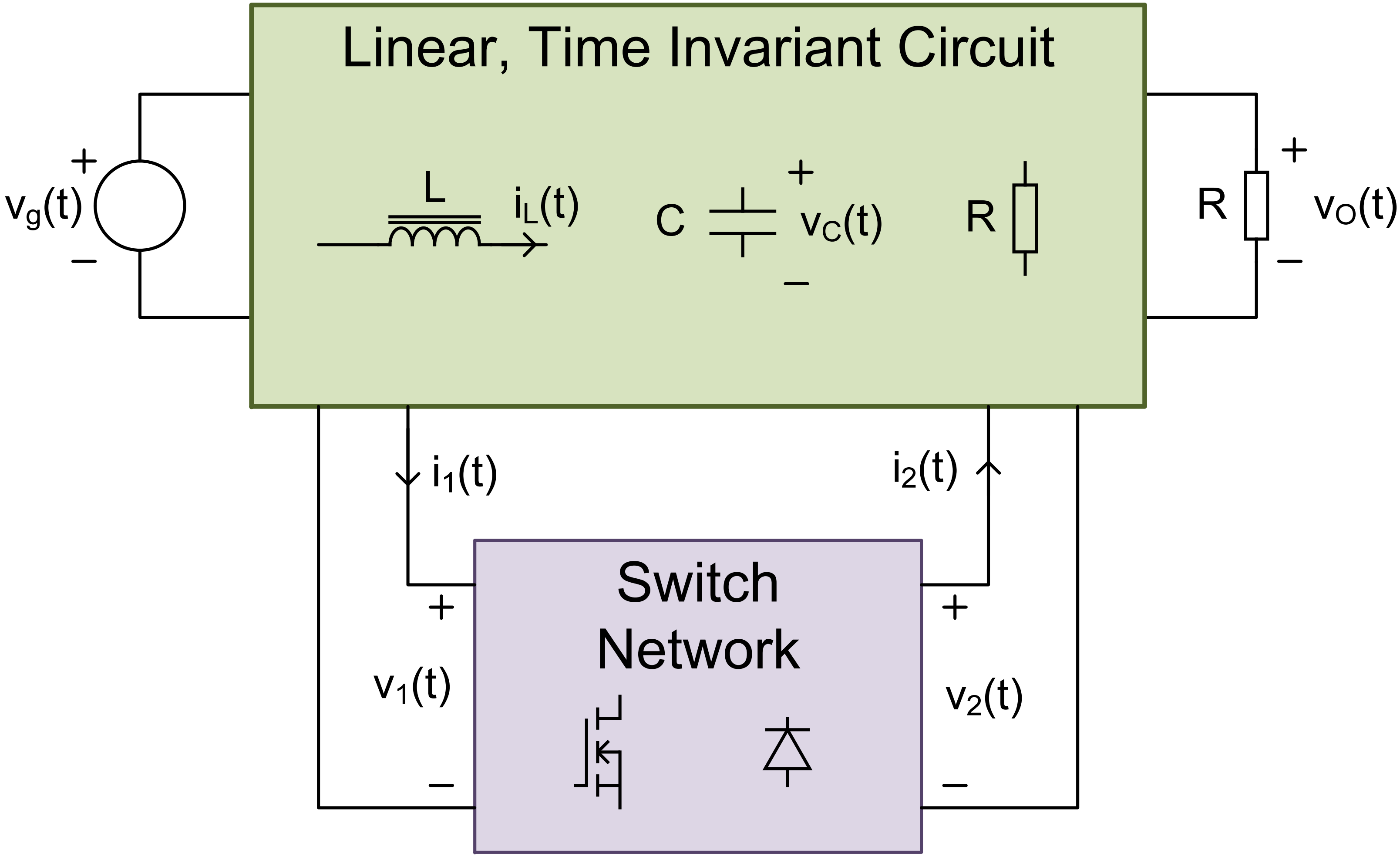
Everything Else Is LTI

Boost And Buck-Boost Switch Networks



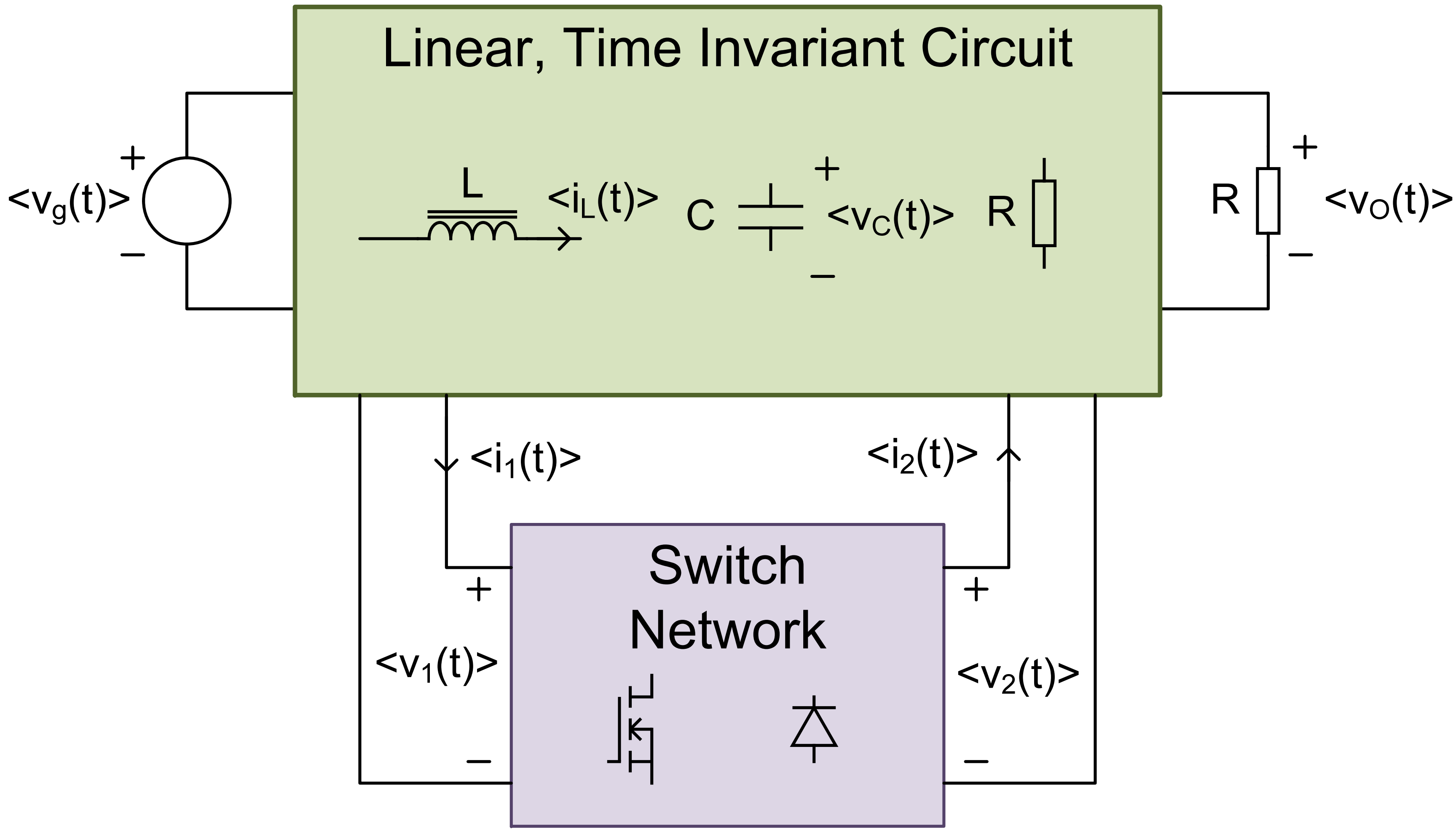


Averaged Switch Modeling

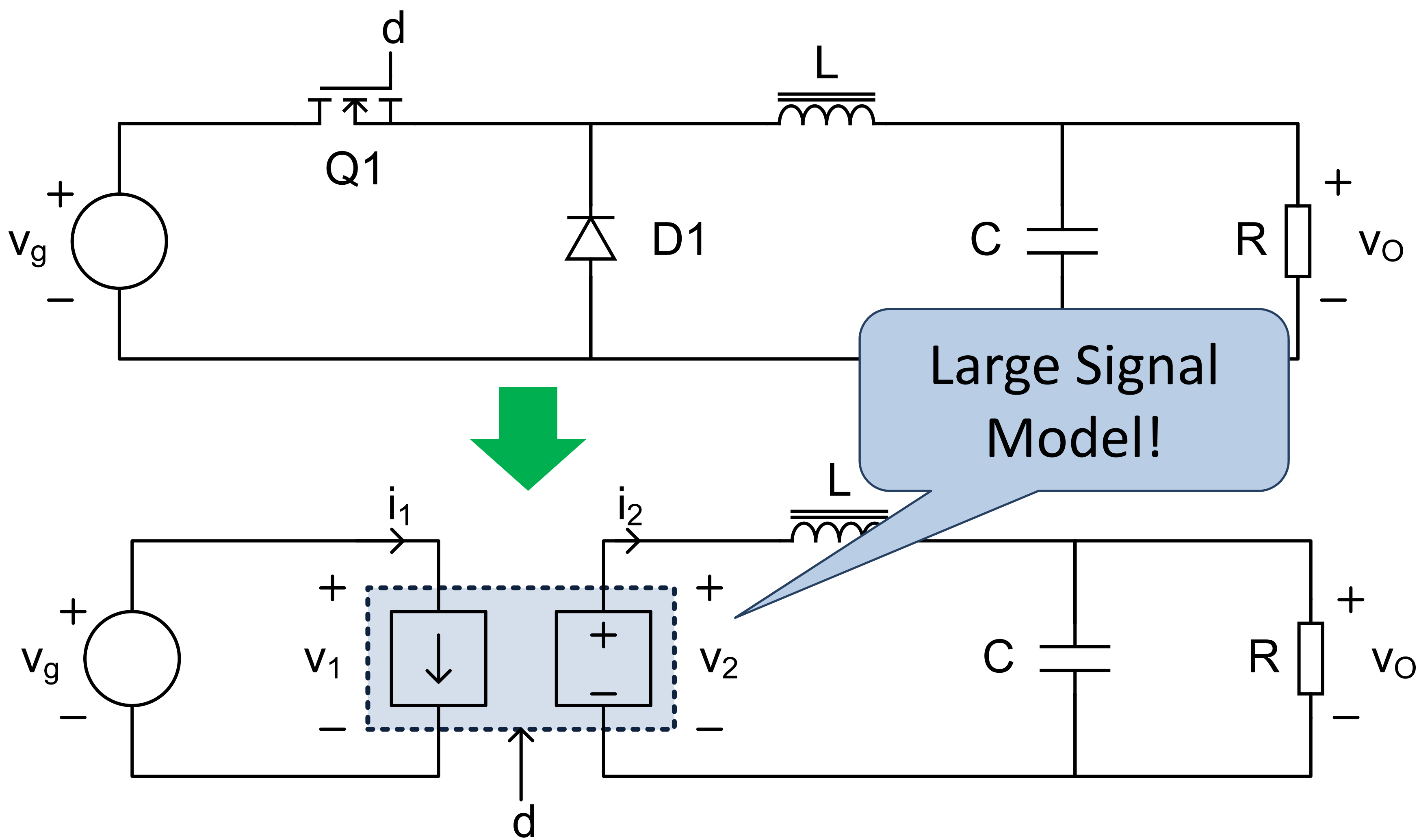


EMBEDDED POWER LABS

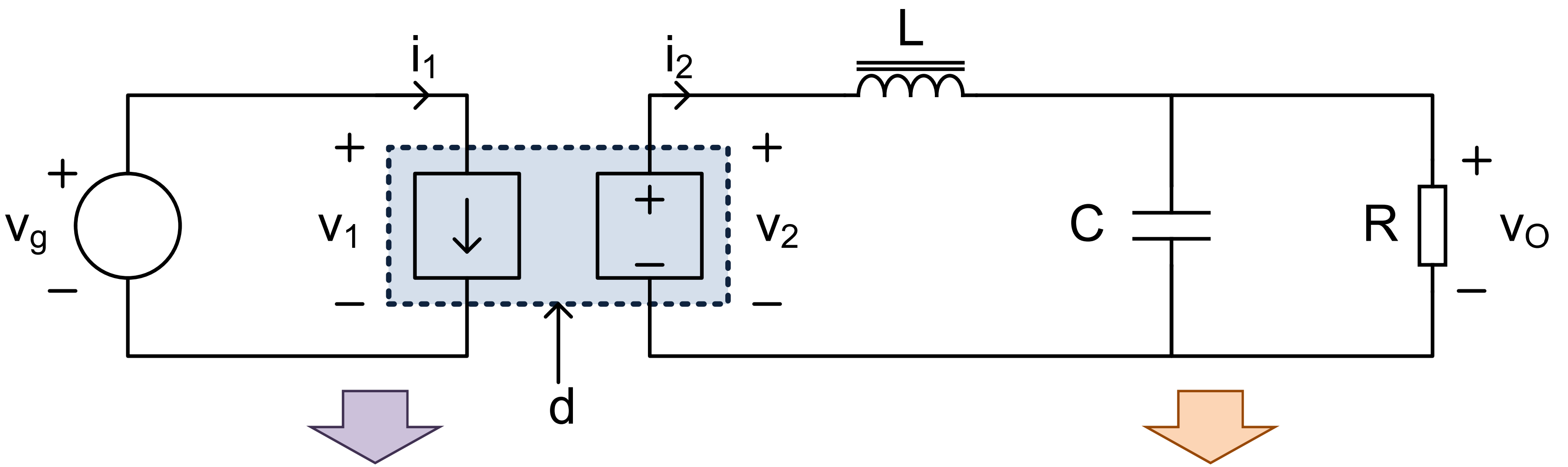
Averaged Switch Modeling



Averaged Switch Modeling



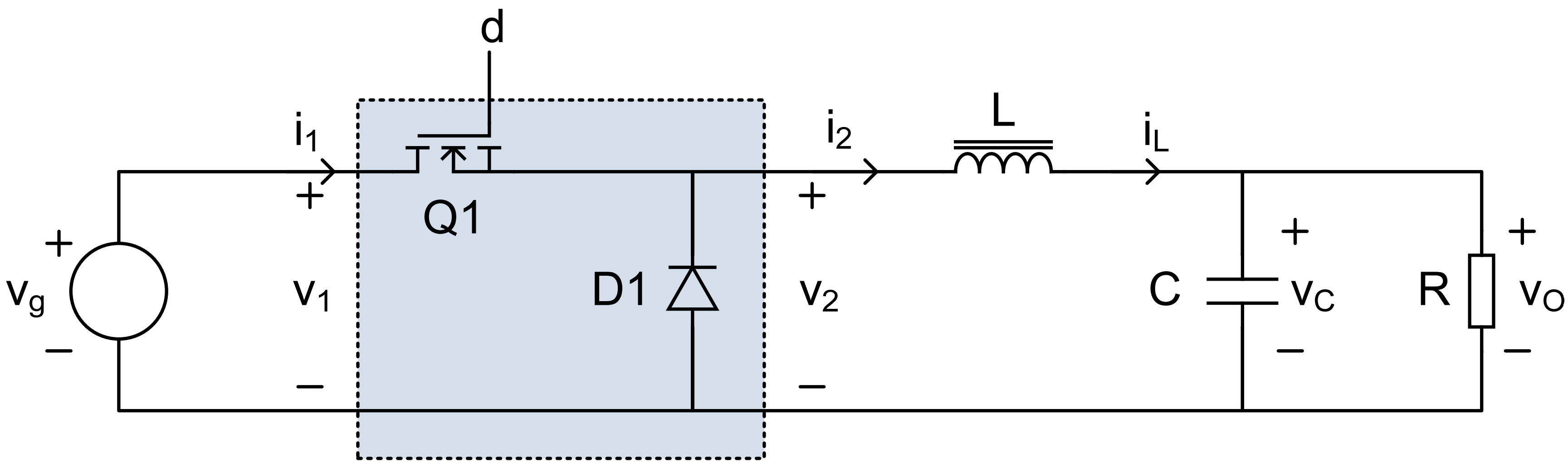
Averaged Switch Modeling



Circuit Simulation
Time Domain
(e.g. Transient Response)
Frequency Domain
(e.g. Bode Plots)

Linearization
Small Signal
Analytical Models
$\Rightarrow$ Transfer Functions

Buck Converter Model: Define Switch Network And Ports

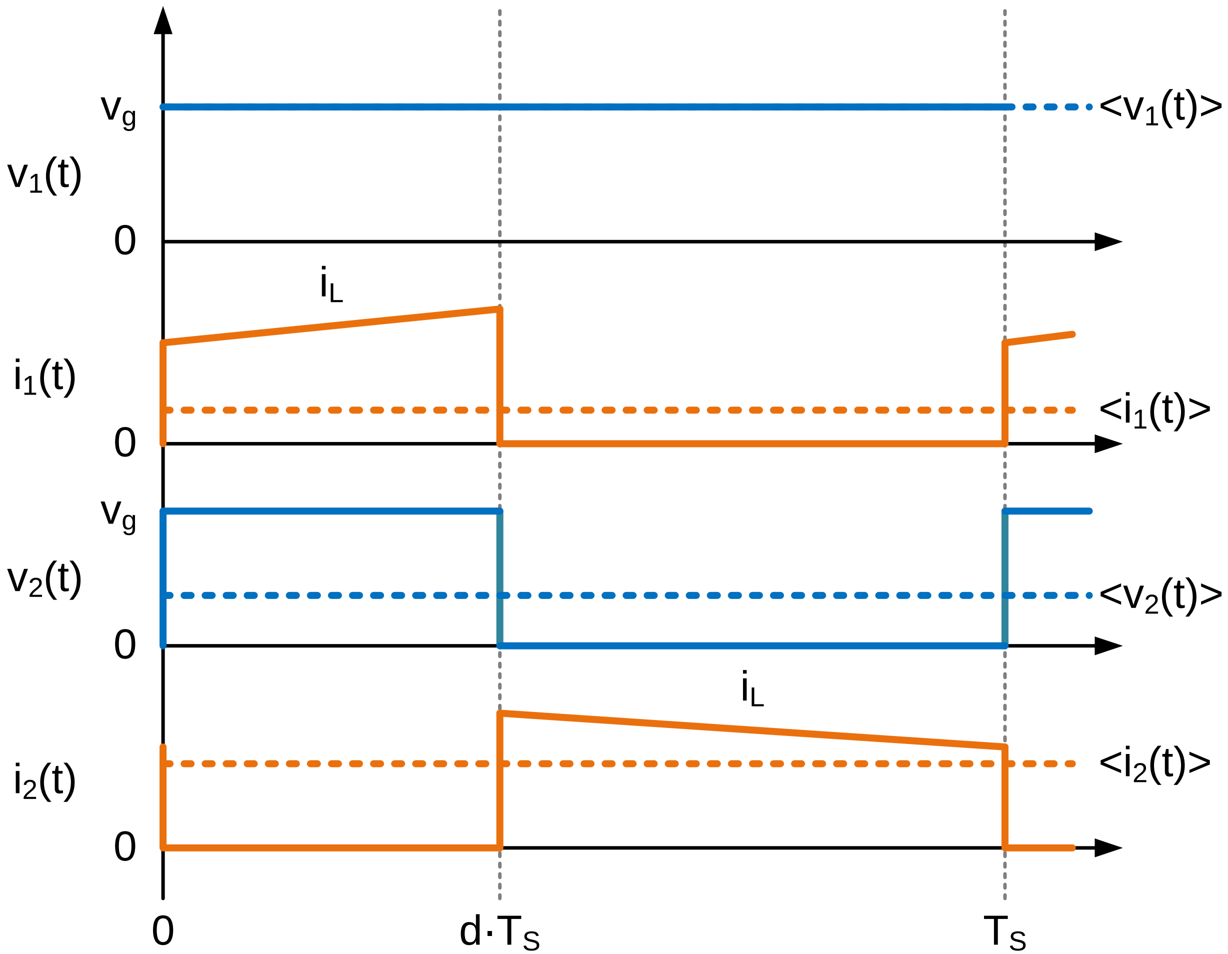


Port 1: v_1, i_1

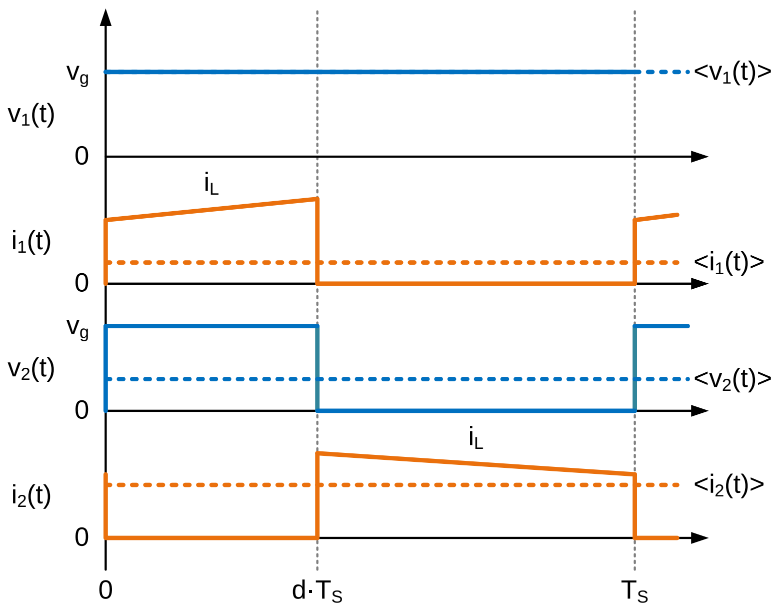
Port 2: v_2, i_2

EMBEDDED POWER LABS

Buck Converter Model: Sketch Waveforms



Buck Converter Model: Average Switch Network Variables



$$\langle v_1(t)\rangle = d\cdot\langle v_g(t)\rangle + d'\cdot\langle v_g(t)\rangle$$
$$= \langle v_g(t)\rangle$$
$$\langle i_1(t)\rangle = d\cdot\langle i_L(t)\rangle + d'\cdot 0$$
$$= d\cdot\langle i_L(t)\rangle$$
$$\langle v_2(t)\rangle = d\cdot\langle v_g(t)\rangle + d'\cdot 0$$
$$= d\cdot\langle v_g(t)\rangle$$
$$\langle i_2(t)\rangle = d\cdot\langle i_L(t)\rangle + d'\cdot\langle i_L(t)\rangle$$
$$= \langle i_L(t)\rangle$$

EMBEDDED POWER LABS

Buck Converter Model: Eliminate Non-Switch Variables

$$\langle v_1(t)\rangle = d\cdot\langle v_g(t)\rangle + d'\cdot\langle v_g(t)\rangle$$

$$= \langle v_g(t)\rangle$$

$$\langle i_1(t)\rangle = d\cdot\langle i_L(t)\rangle + d'\cdot 0$$

$$= d\cdot\langle i_L(t)\rangle$$

$$\langle v_2(t)\rangle = d\cdot\langle v_g(t)\rangle + d'\cdot 0$$

$$= d\cdot\langle v_g(t)\rangle$$

$$\langle i_2(t)\rangle = d\cdot\langle i_L(t)\rangle + d'\cdot\langle i_L(t)\rangle$$

$$= \langle i_L(t)\rangle$$

$$\langle v_2(t)\rangle = d\cdot\langle v_g(t)\rangle = d\cdot\langle v_1(t)\rangle$$

EMBEDDED POWER LABS

Buck Converter Model: Eliminate Non-Switch Variables

$$\langle v_1(t)\rangle = d\cdot\langle v_g(t)\rangle + d'\cdot\langle v_g(t)\rangle$$

$$= \langle v_g(t)\rangle$$

$$\langle i_1(t)\rangle = d\cdot\langle i_L(t)\rangle + d'\cdot 0$$

$$= d\cdot\langle i_L(t)\rangle$$

$$\langle v_2(t)\rangle = d\cdot\langle v_g(t)\rangle + d'\cdot 0$$

$$= d\cdot\langle v_g(t)\rangle$$

$$\langle i_2(t)\rangle = d\cdot\langle i_L(t)\rangle + d'\cdot\langle i_L(t)\rangle$$

$$= \langle i_L(t)\rangle$$

$$\langle v_2(t)\rangle = d\cdot\langle v_g(t)\rangle = d\cdot\langle v_1(t)\rangle$$

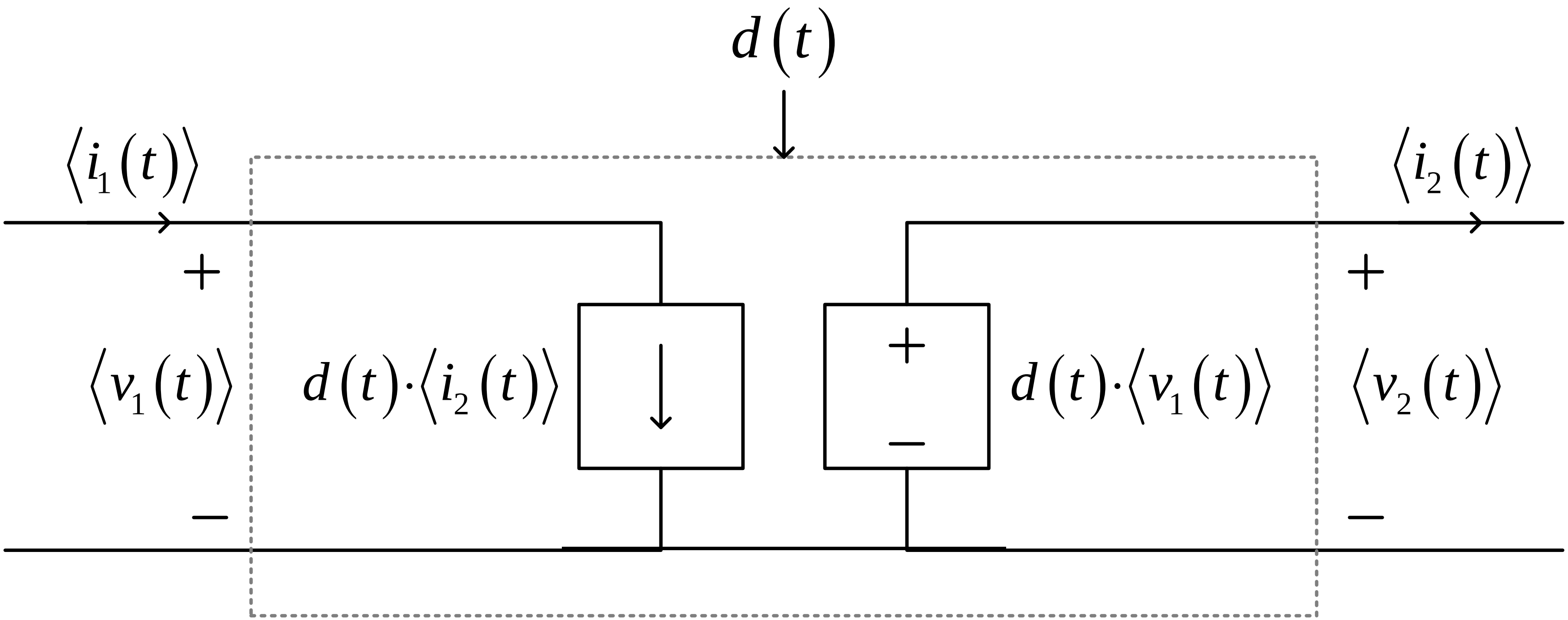
$$\langle i_1(t)\rangle = d\cdot\langle i_L(t)\rangle = d\cdot\langle i_2(t)\rangle$$

EMBEDDED POWER LABS

Buck Converter Model: Create Switch Network Model

!

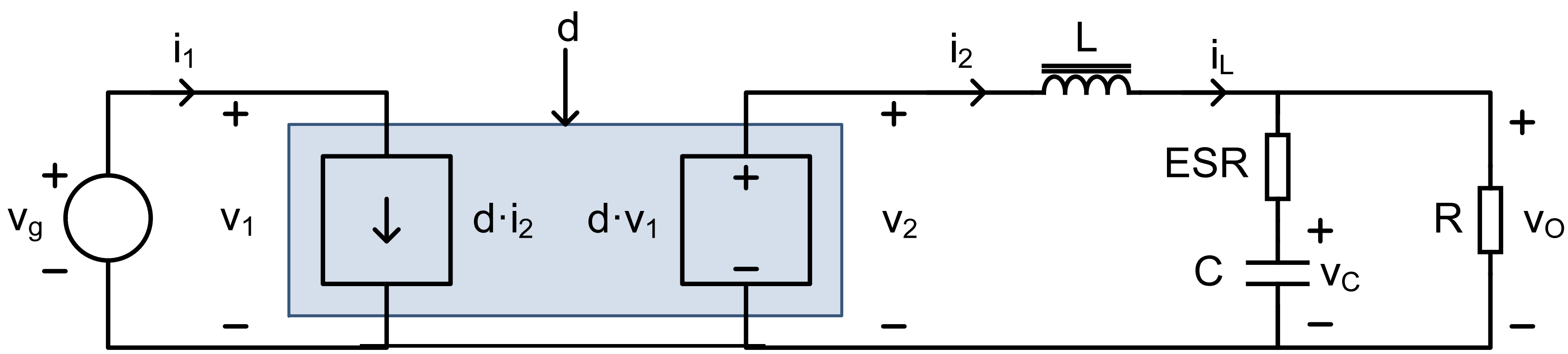
$$\langle i_1(t)\rangle = d(t)\cdot\langle i_2(t)\rangle \qquad \langle v_2(t)\rangle = d(t)\cdot\langle v_1(t)\rangle$$



EMBEDDED POWER LABS

Buck Converter Model: Complete The Converter Model

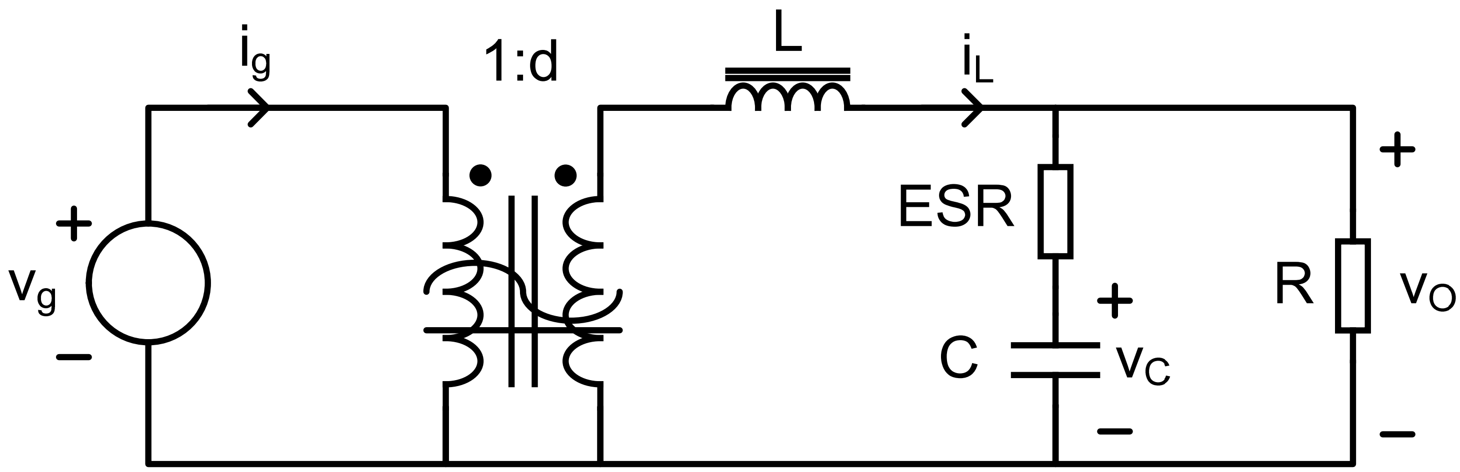
!



EMBEDDED POWER LABS

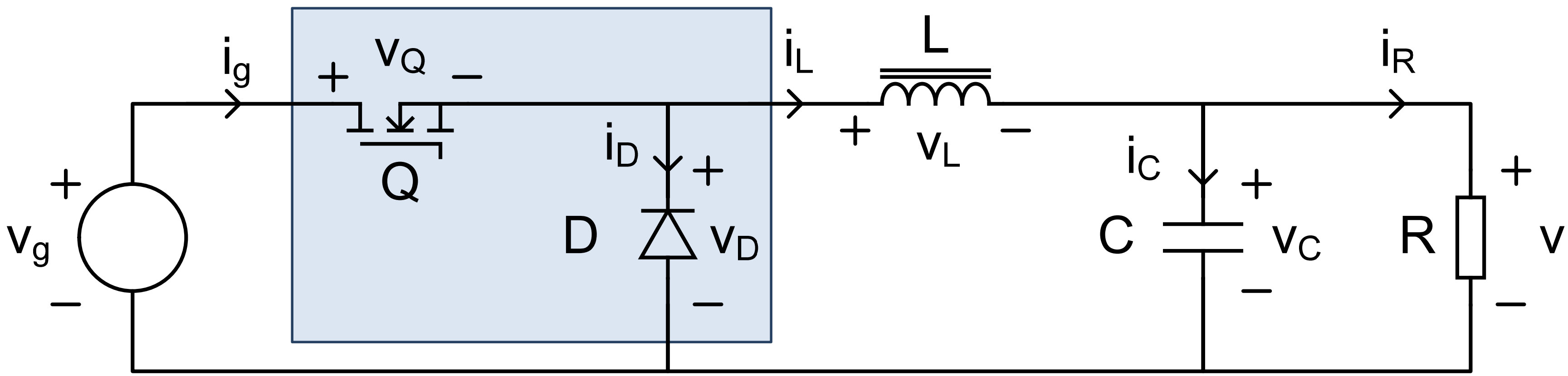
Buck Converter Model: Complete The Converter Model

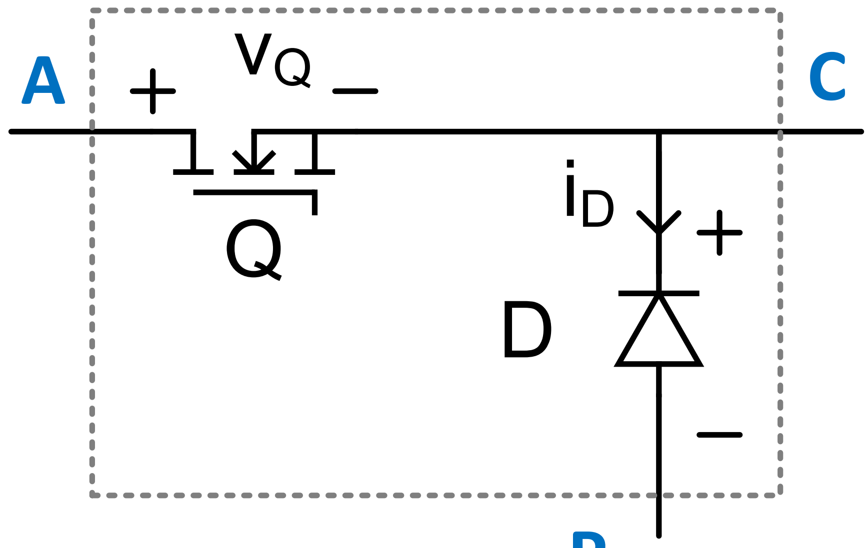
!



EMBEDDED POWER LABS

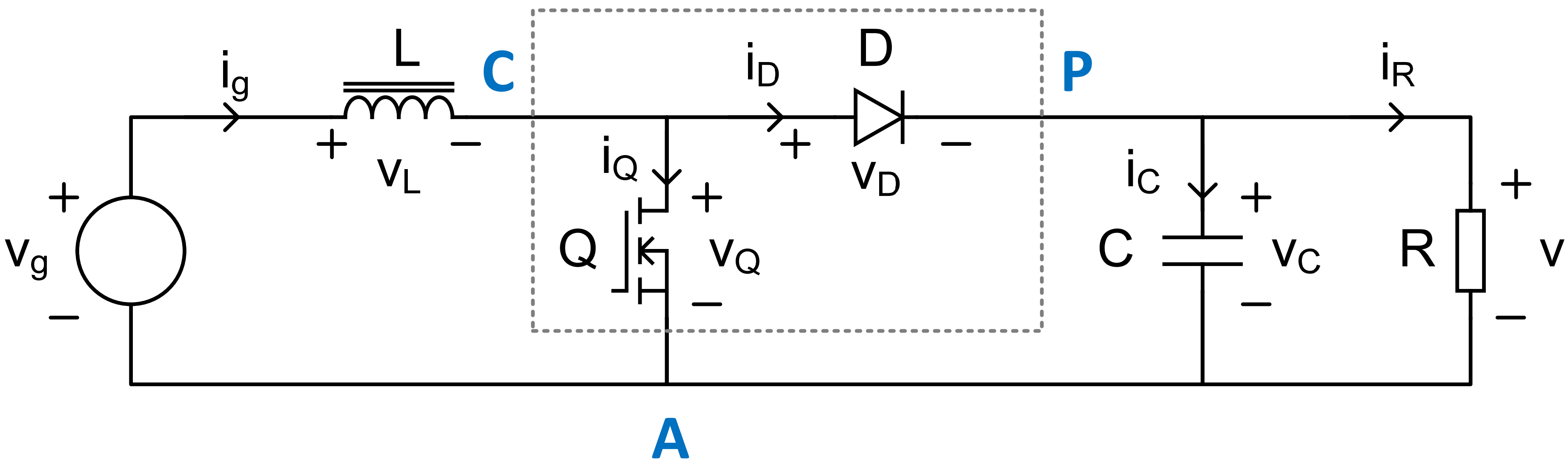
Another View Of The Average Switch Model



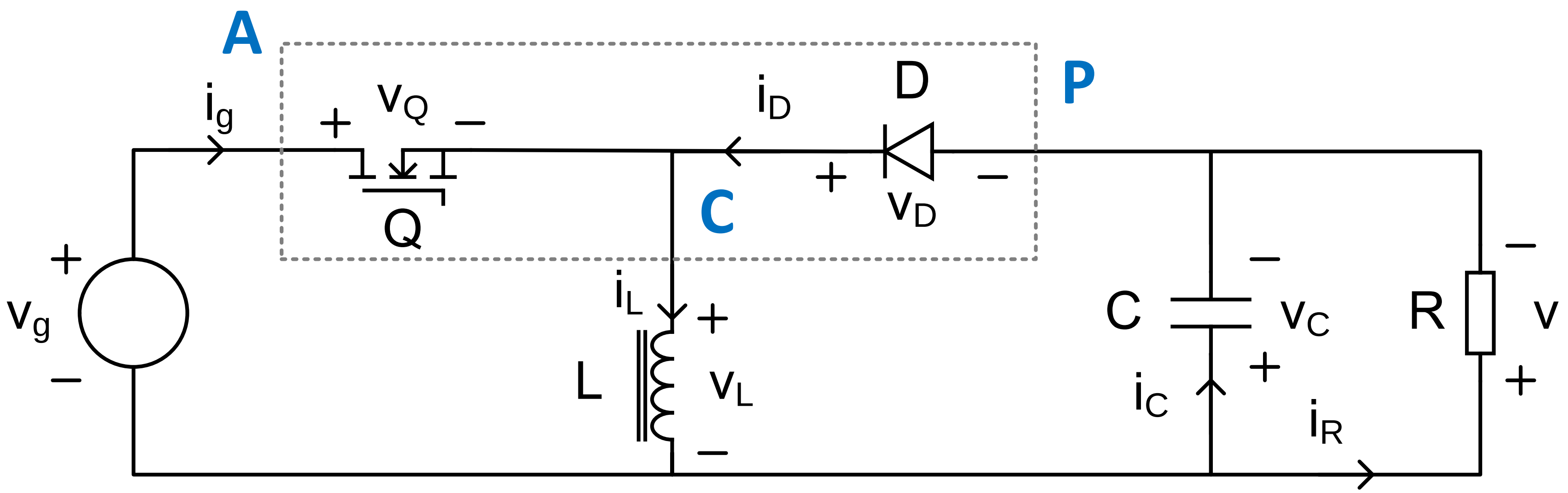


EMBEDDED POWER LABS

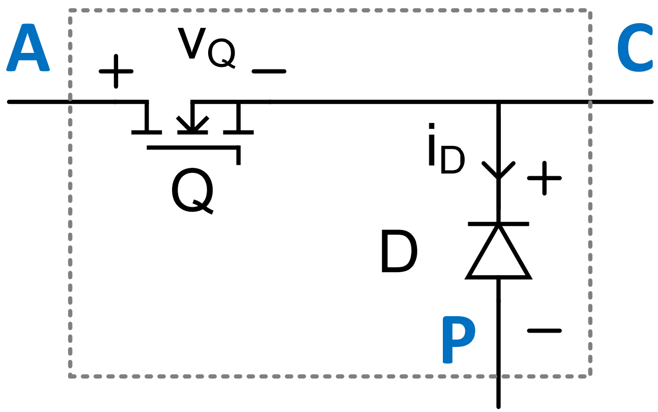
Another View Of The Average Switch Model

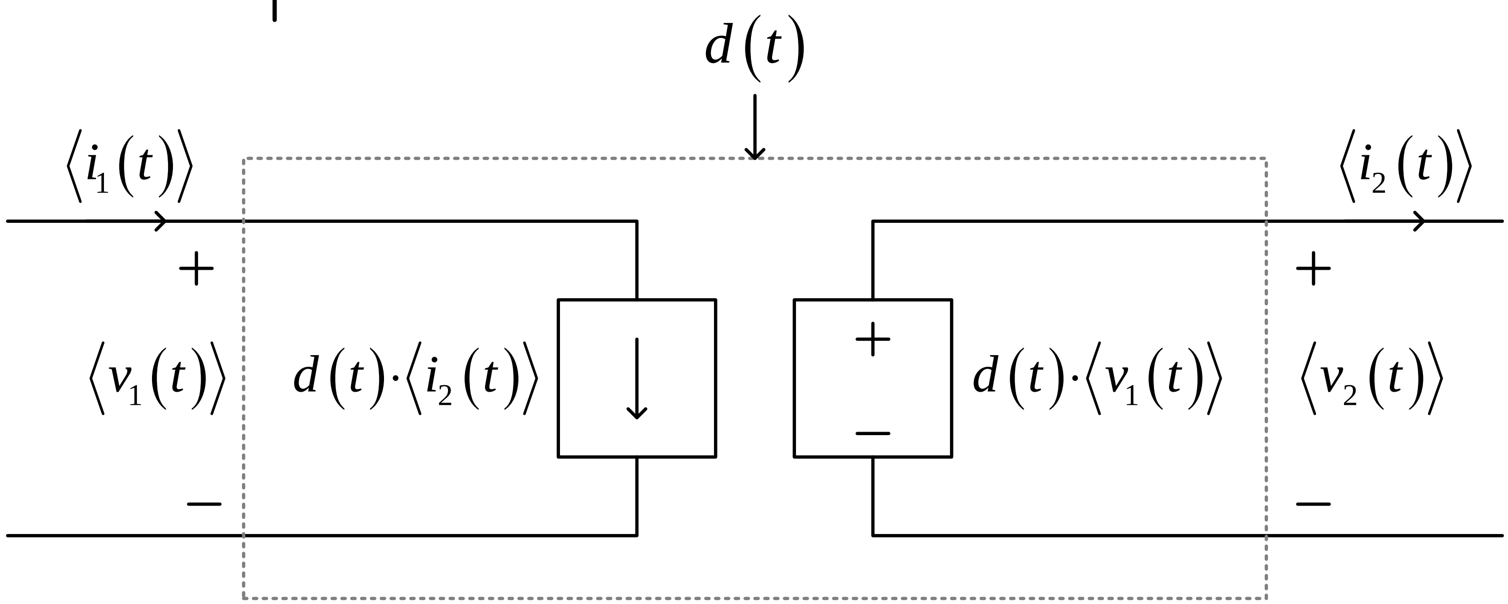


Another View Of The Average Switch Model

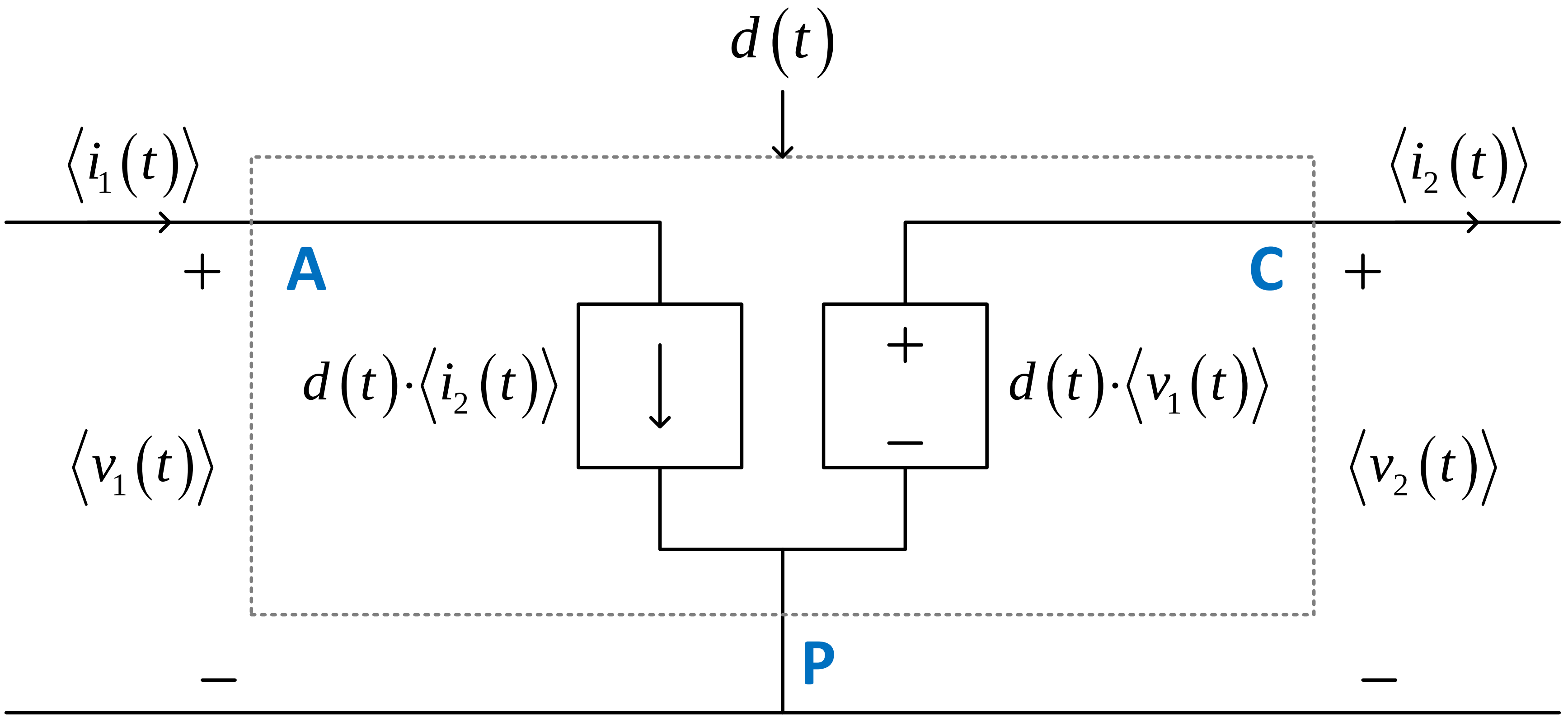


Another View Of The Average Switch Model



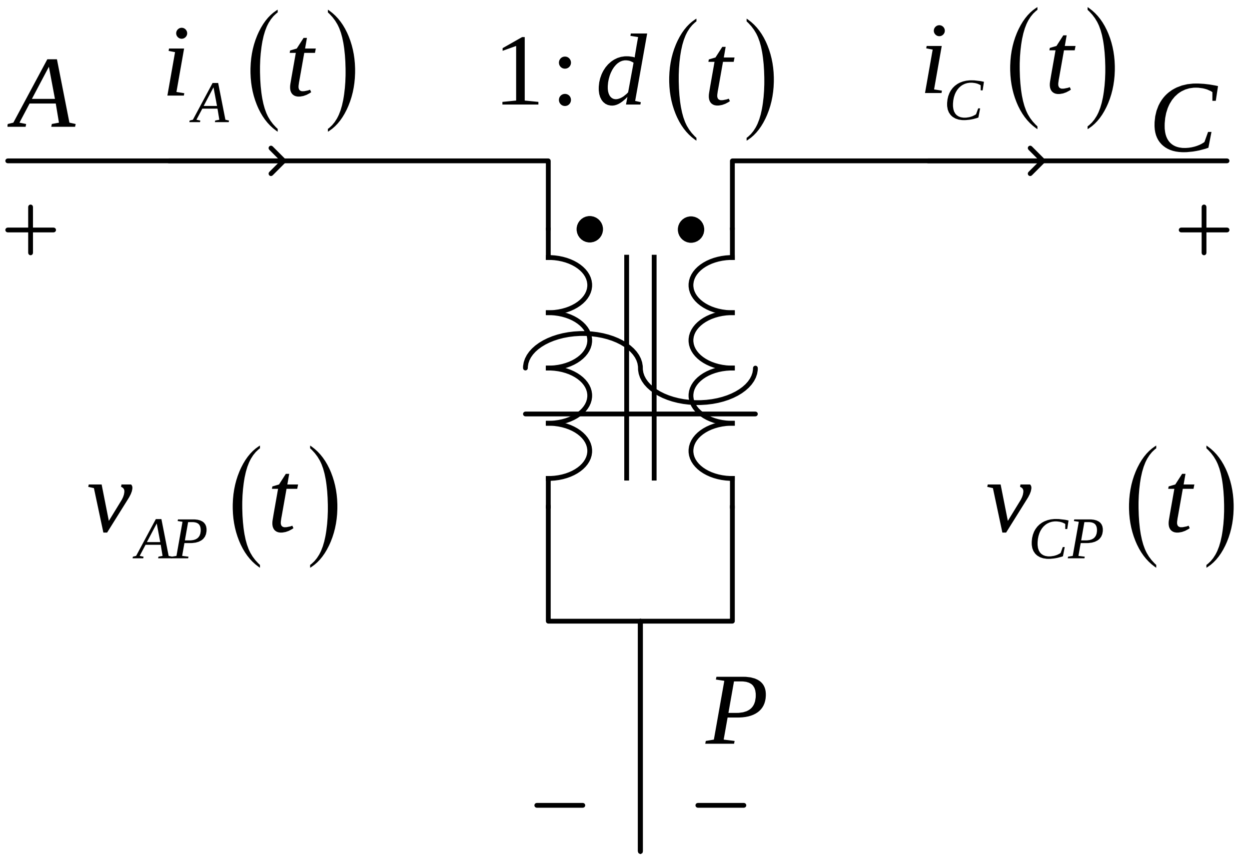


Another View Of The Average Switch Model



EMBEDDED POWER LABS

Another View Of The Average Switch Model



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Simulating Buck Converter With Averaged Switch Model

- F_{SWITCH} = 500 kHz
- V_g = 12 Vdc
- V_O = 3 Vdc
- D = 0.25
- I_O = 3 A
- R_O = 1 Ω
- ΔI_L = 20% I_O = 0.6 A ⇒ L = 7.5 μH
- C = 33 μF
  - ESR = 50 mΩ
  - F_{ZERO} = 96.5 kHz
  - ΔV_C= 9.1 mV (capacitor only)
- F_0 = 10.1 kHz
- Q = 2.1 = 6.4 dB

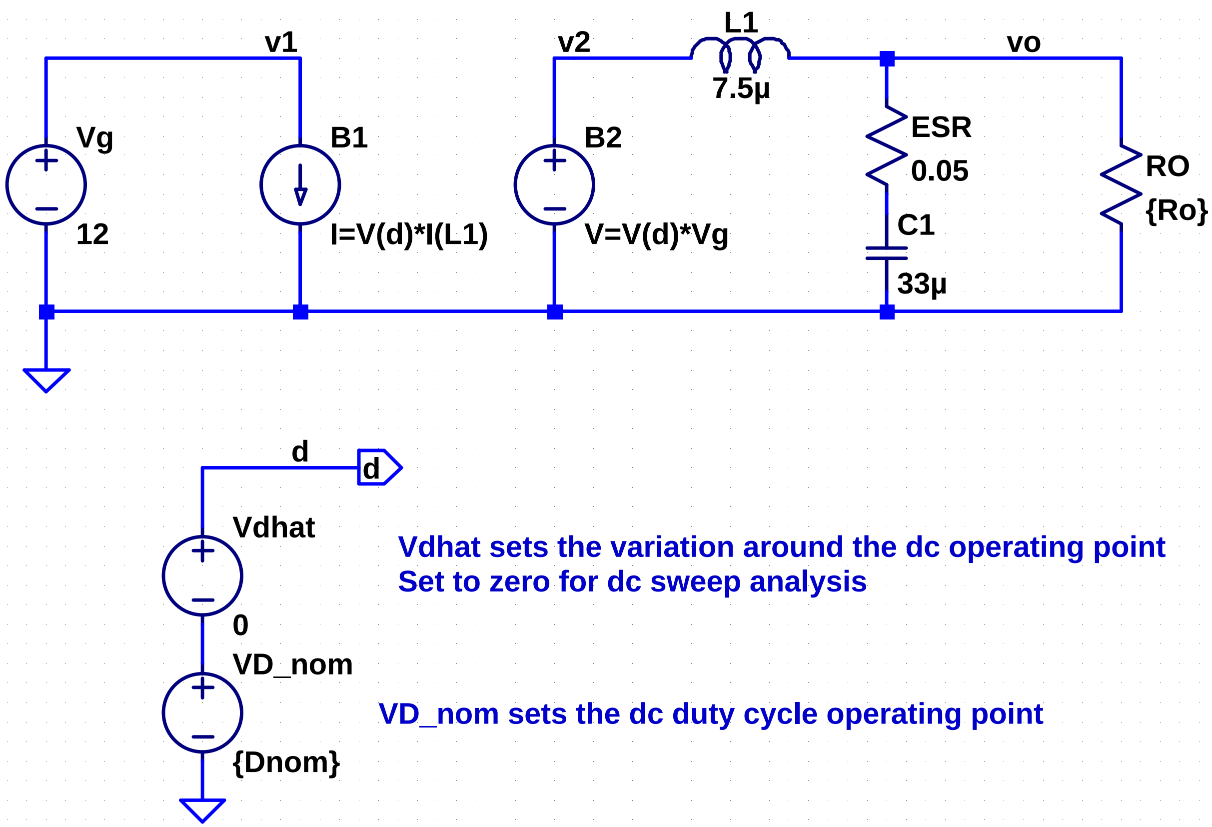
LTspice Model: DC Sweep

Input Parameters (Vg, Vo_nom, Ro) Calculated Parameters (Io_nom, Dnom)

.param Vg = 12
.param Vo_nom = 3
.param Ro = 1

.param Dnom = (Vo_nom/Vg)
.param Io_nom = Vo_nom/Ro

.dc VD_nom 0 0.95 0.01



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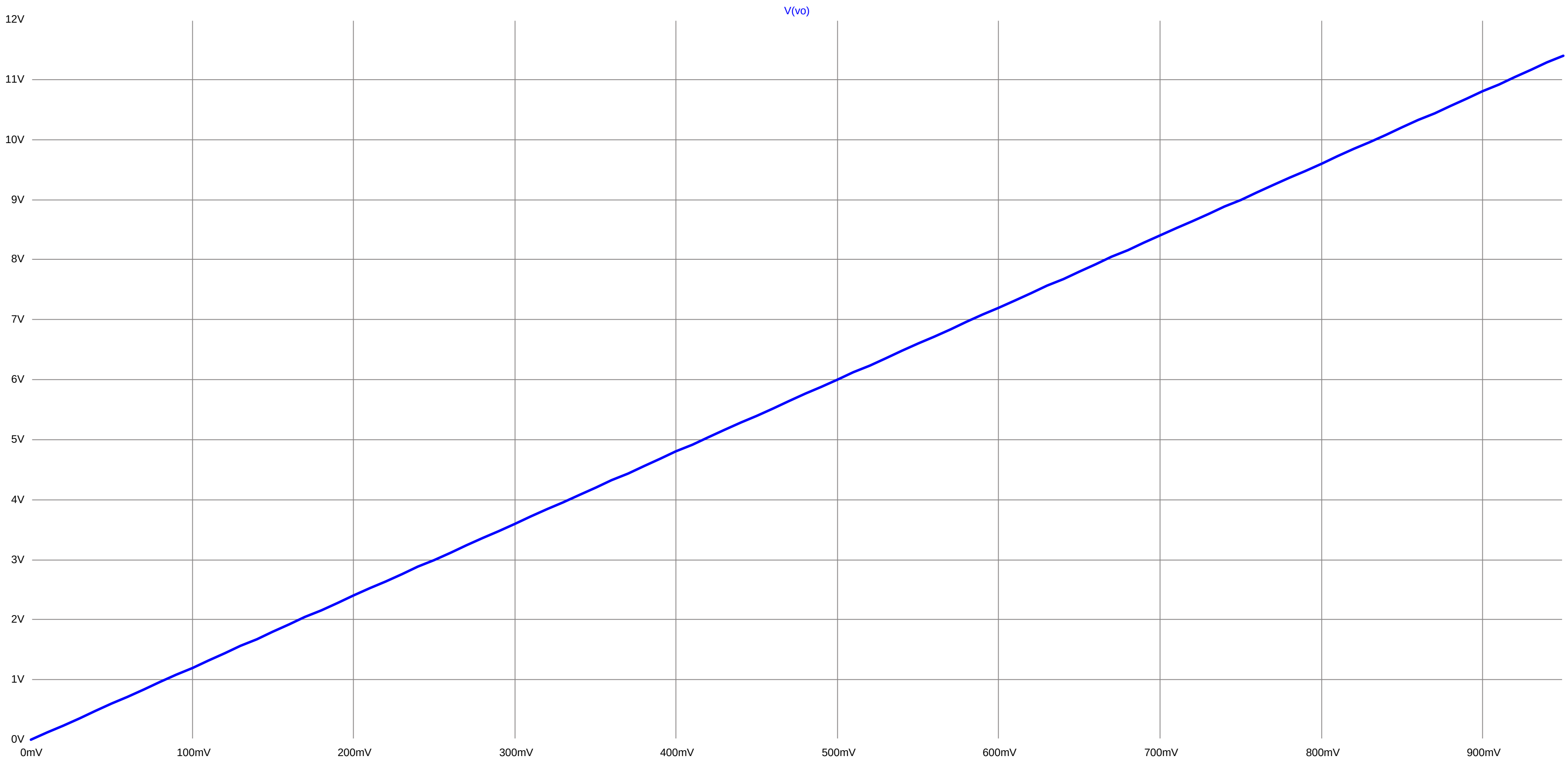
LTspice Model: DC Sweep



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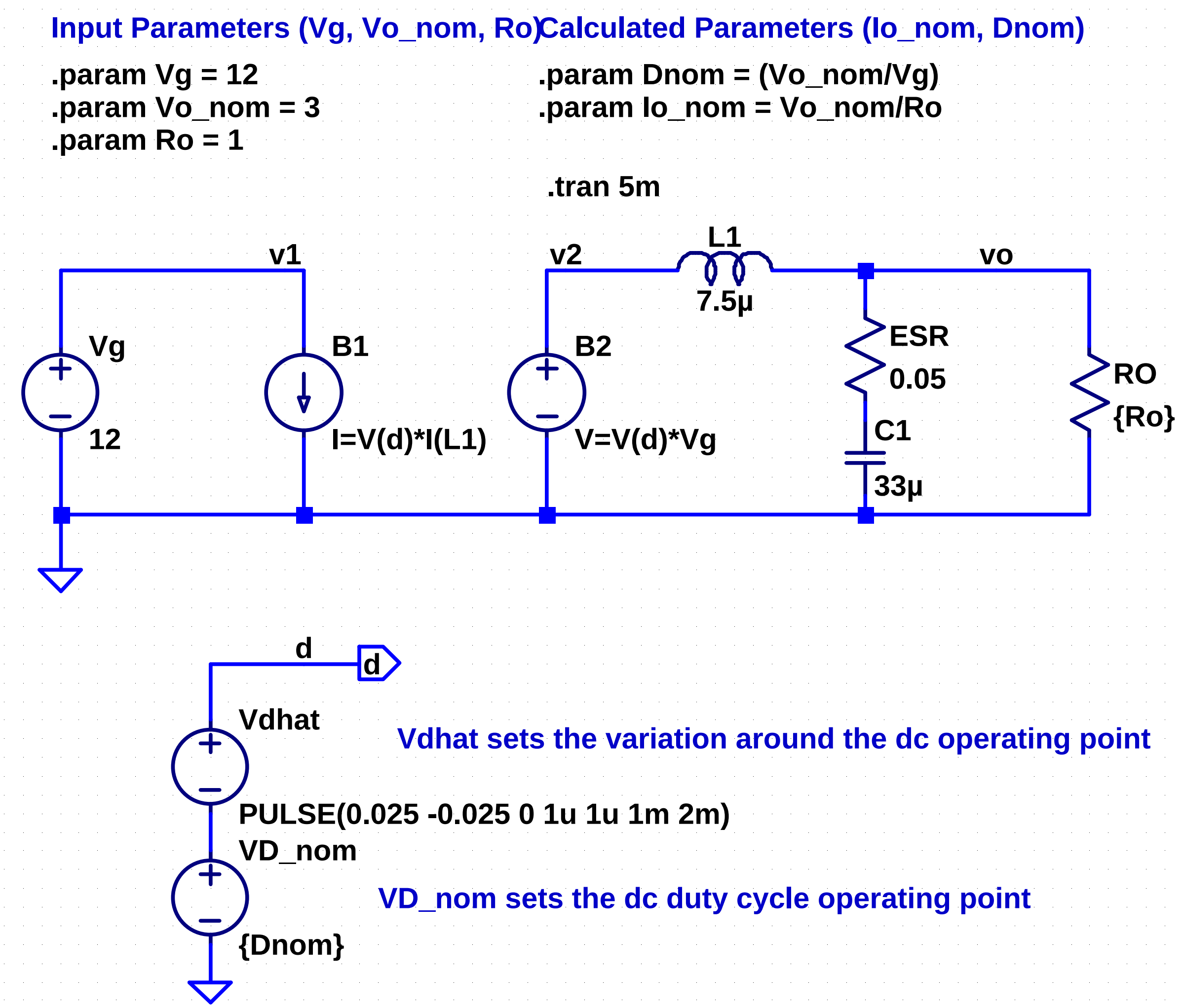
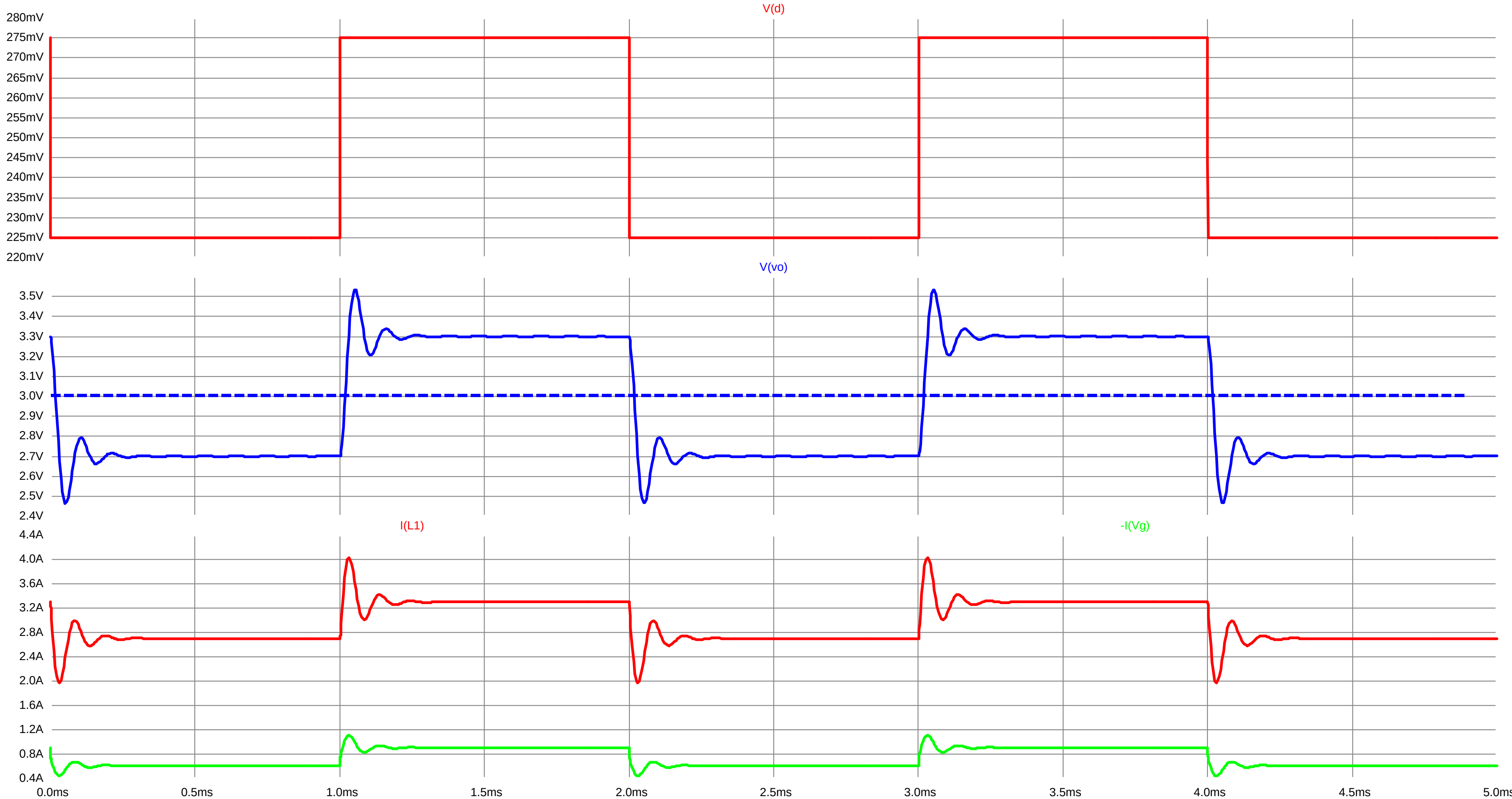
LTspice Model: Transient Response



LTspice Transient Simulation



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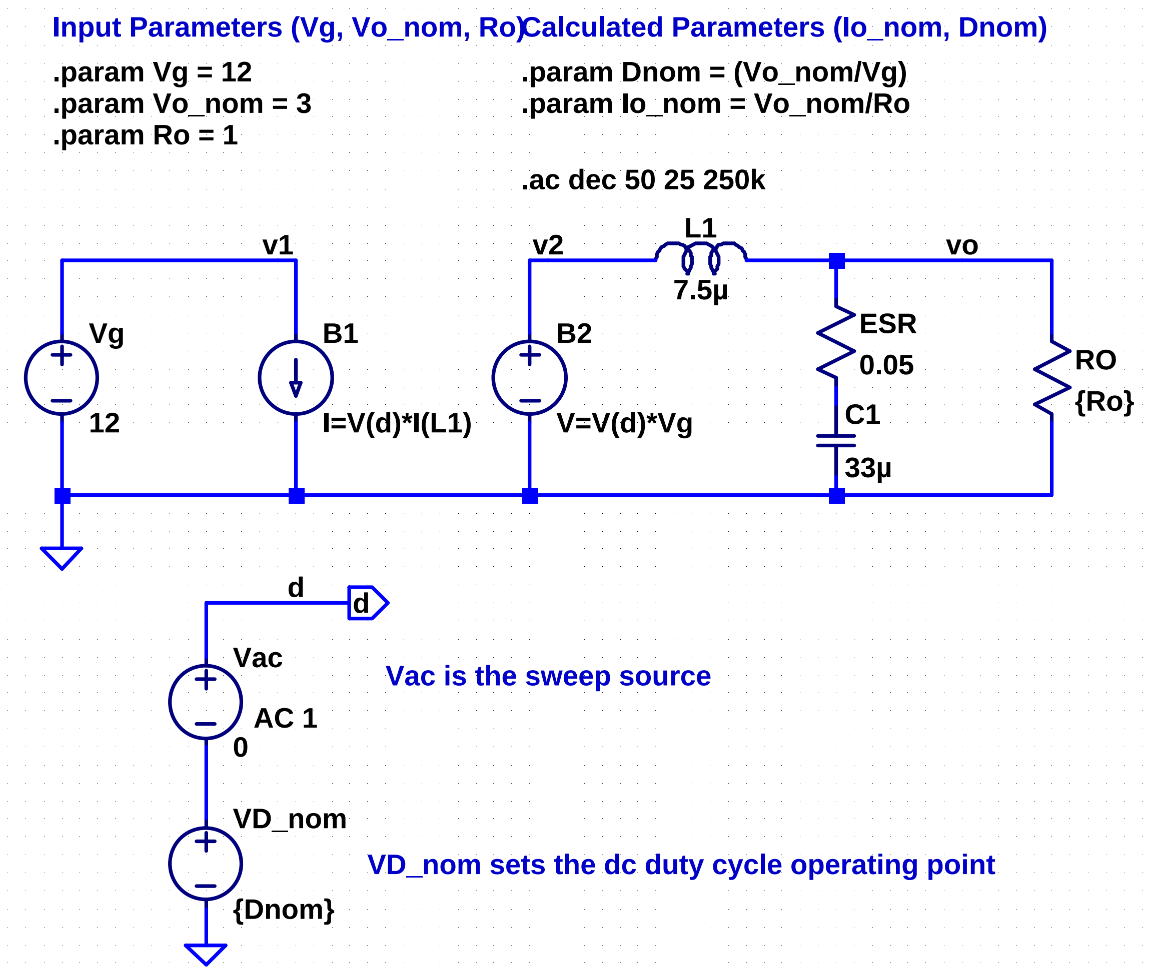
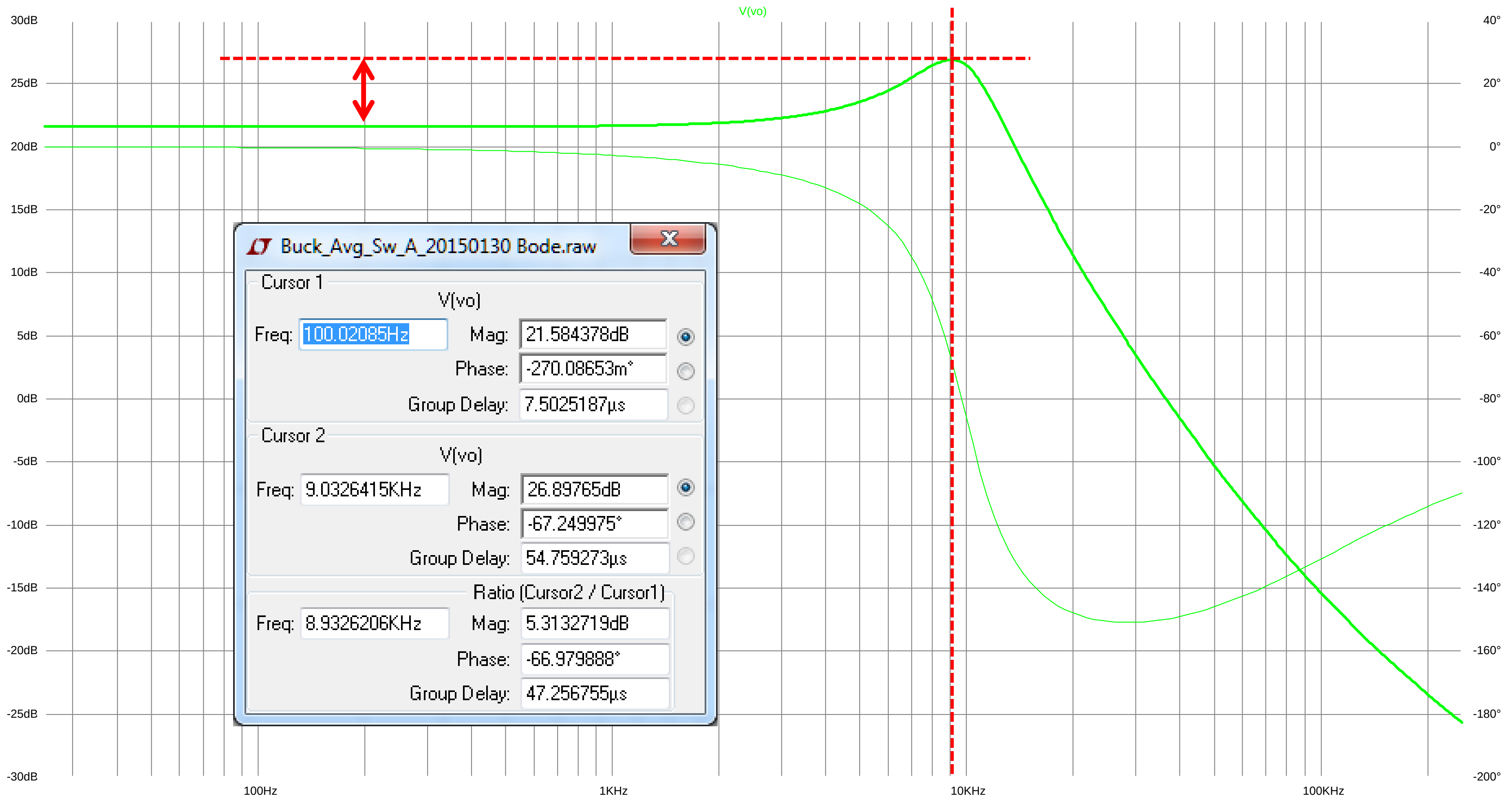
LTspice Model: Plotting G_{vd}

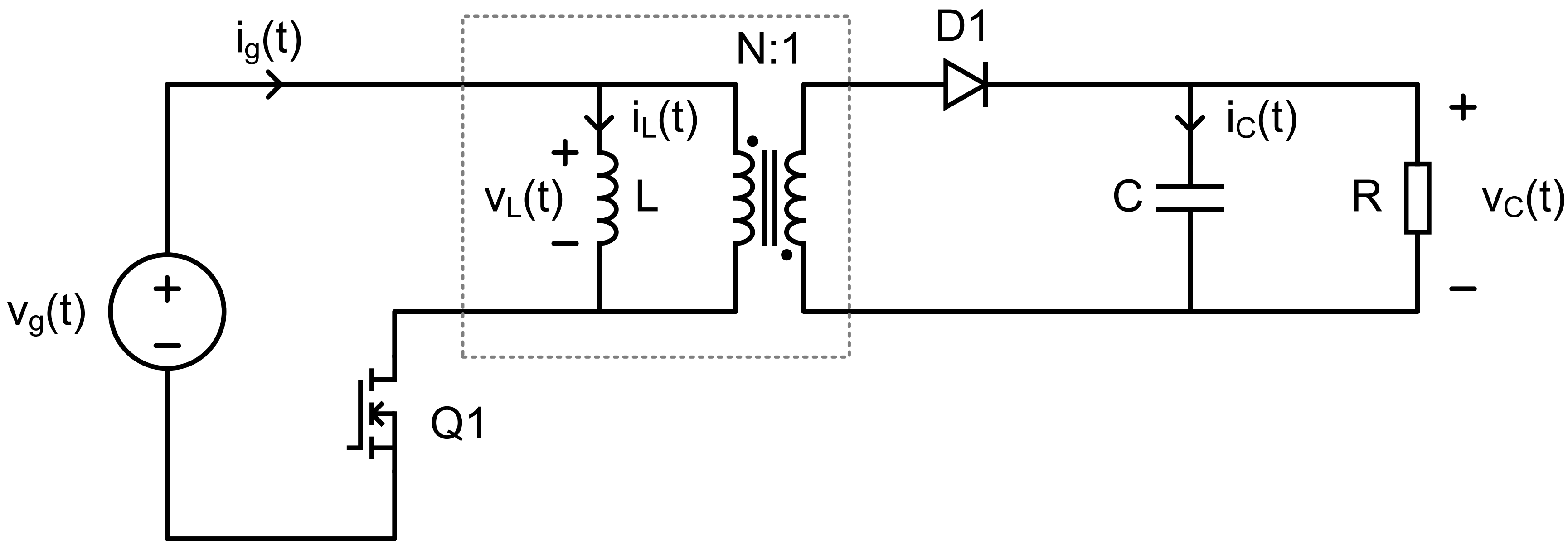


LTspice Model

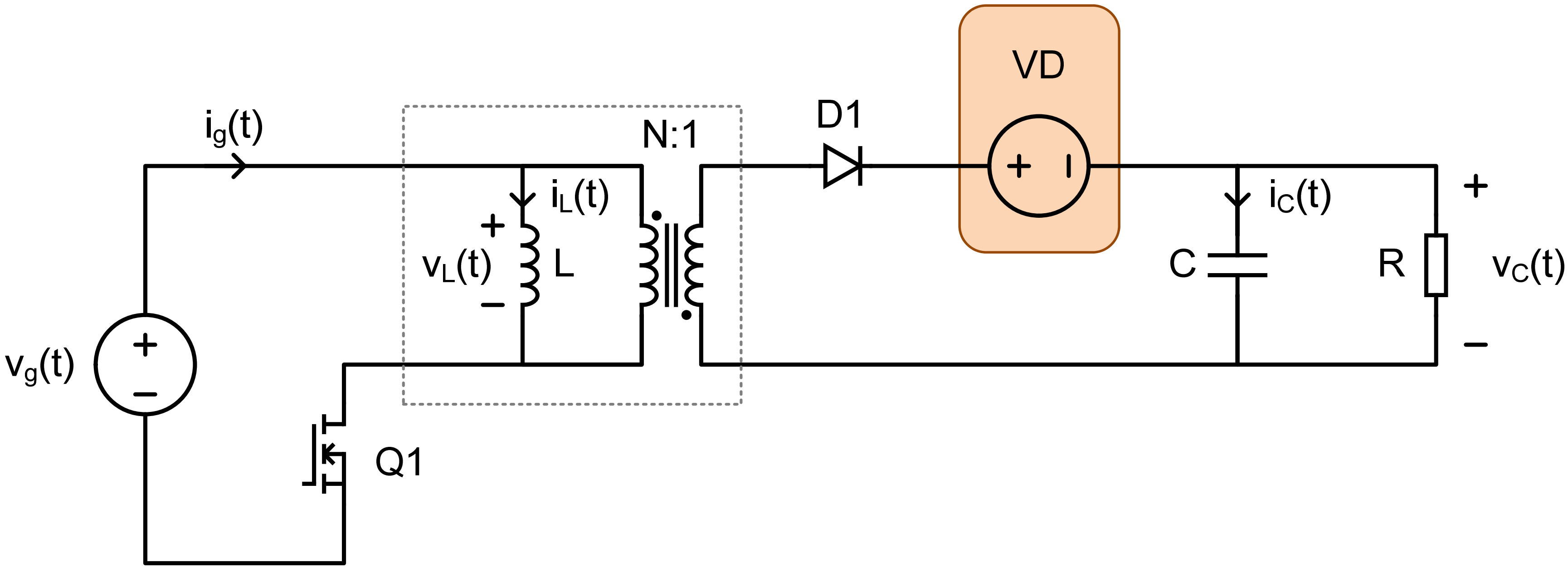


Embedded Power Labs

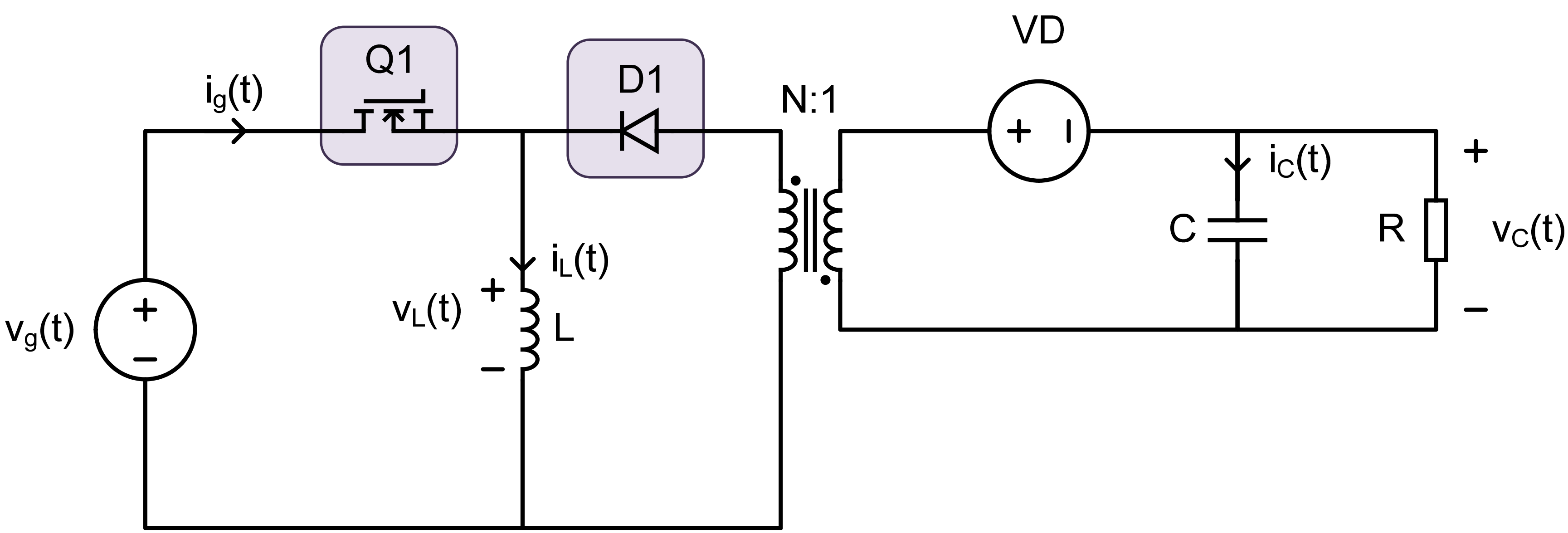
Flyback Simulation With Average Switch Model



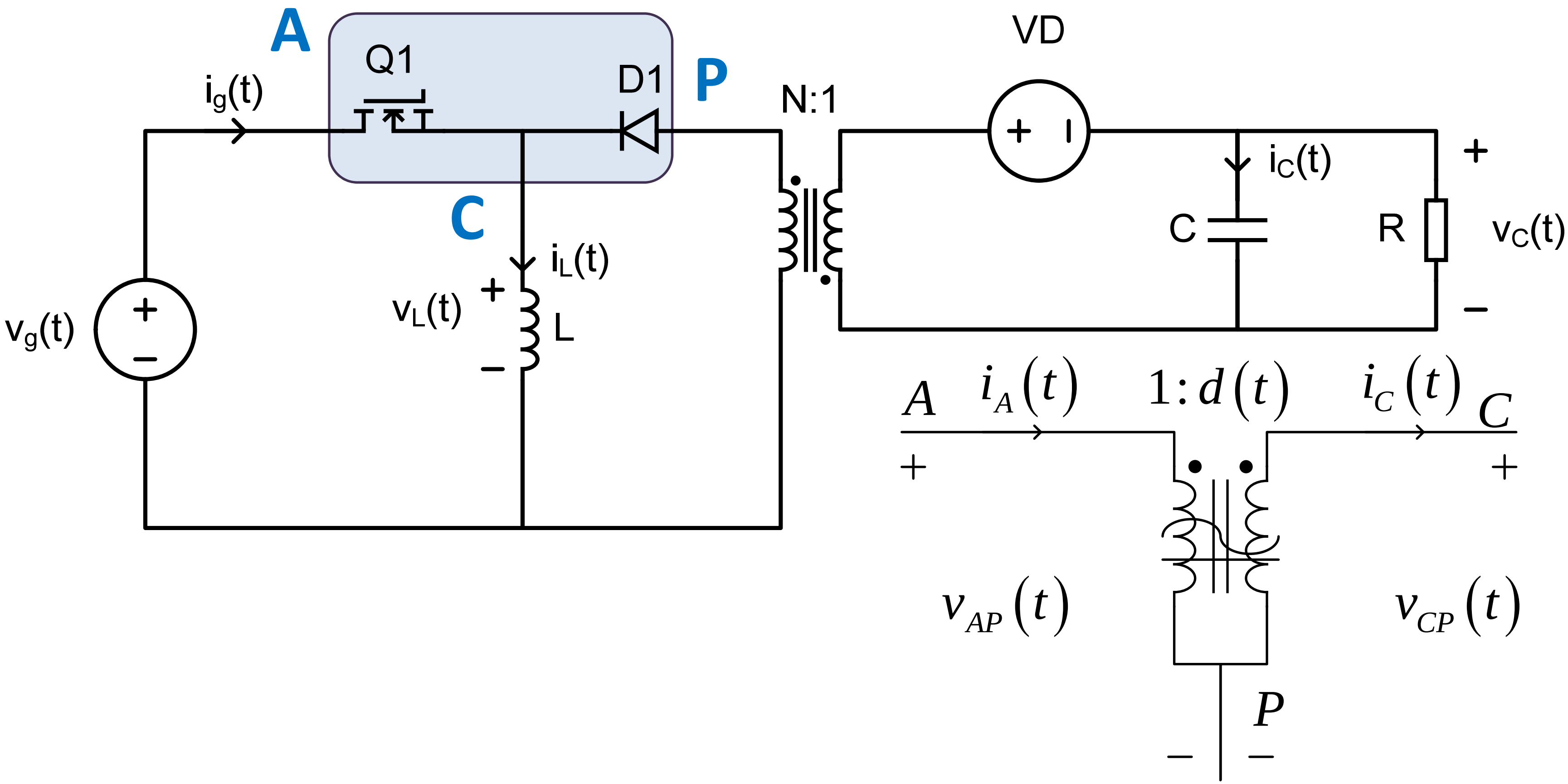
Flyback Simulation With Average Switch Model



Flyback Simulation With Average Switch Model

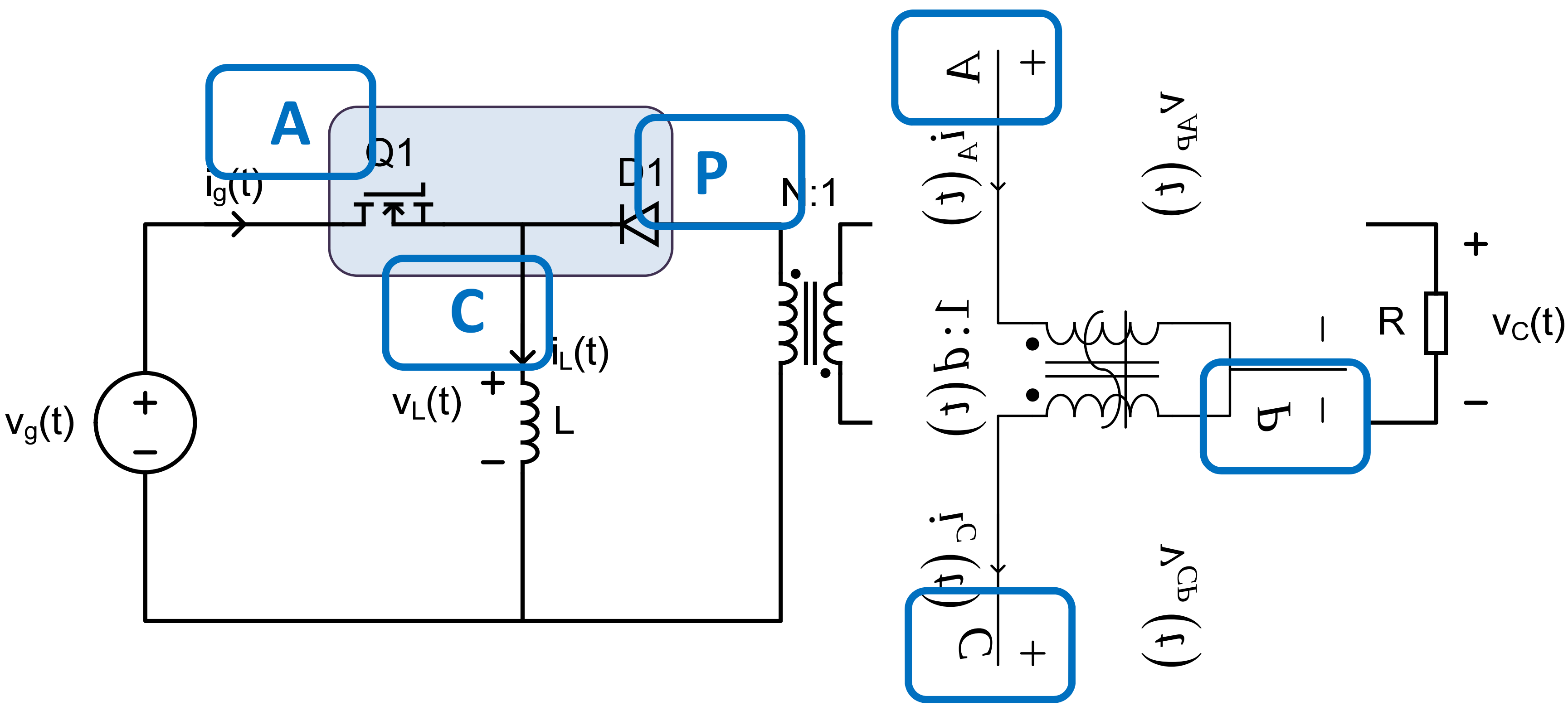


Flyback Simulation With Average Switch Model

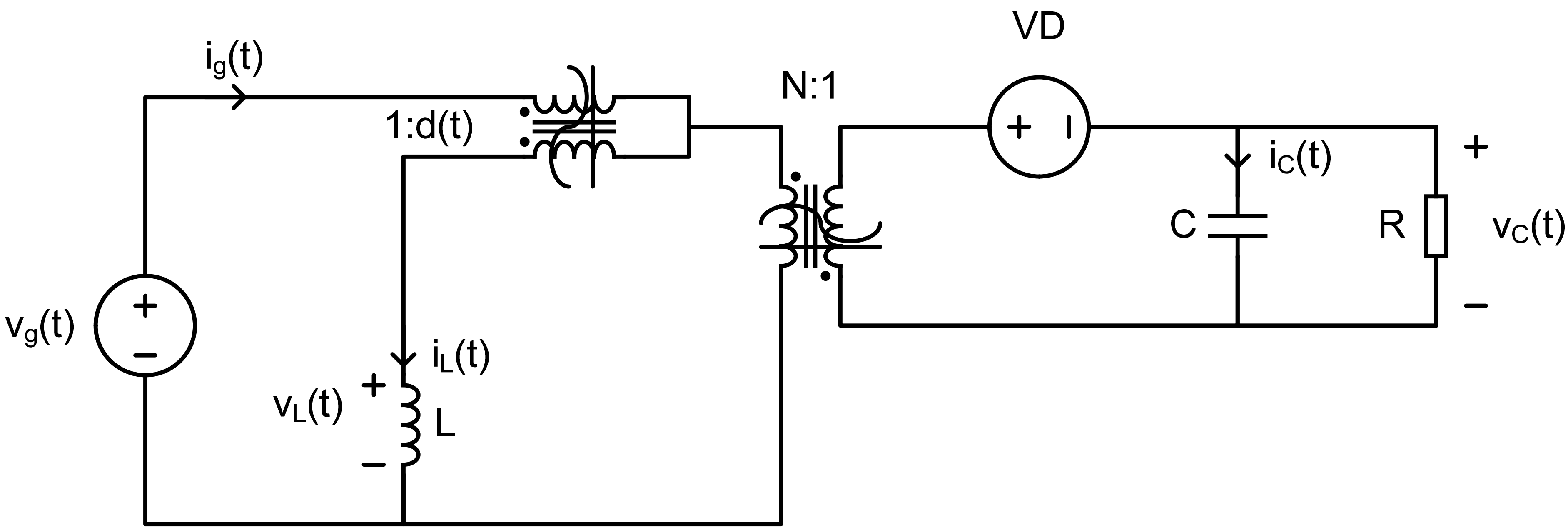


Embedded Power Labs

Flyback Simulation With Average Switch Model



Flyback Simulation With Average Switch Model



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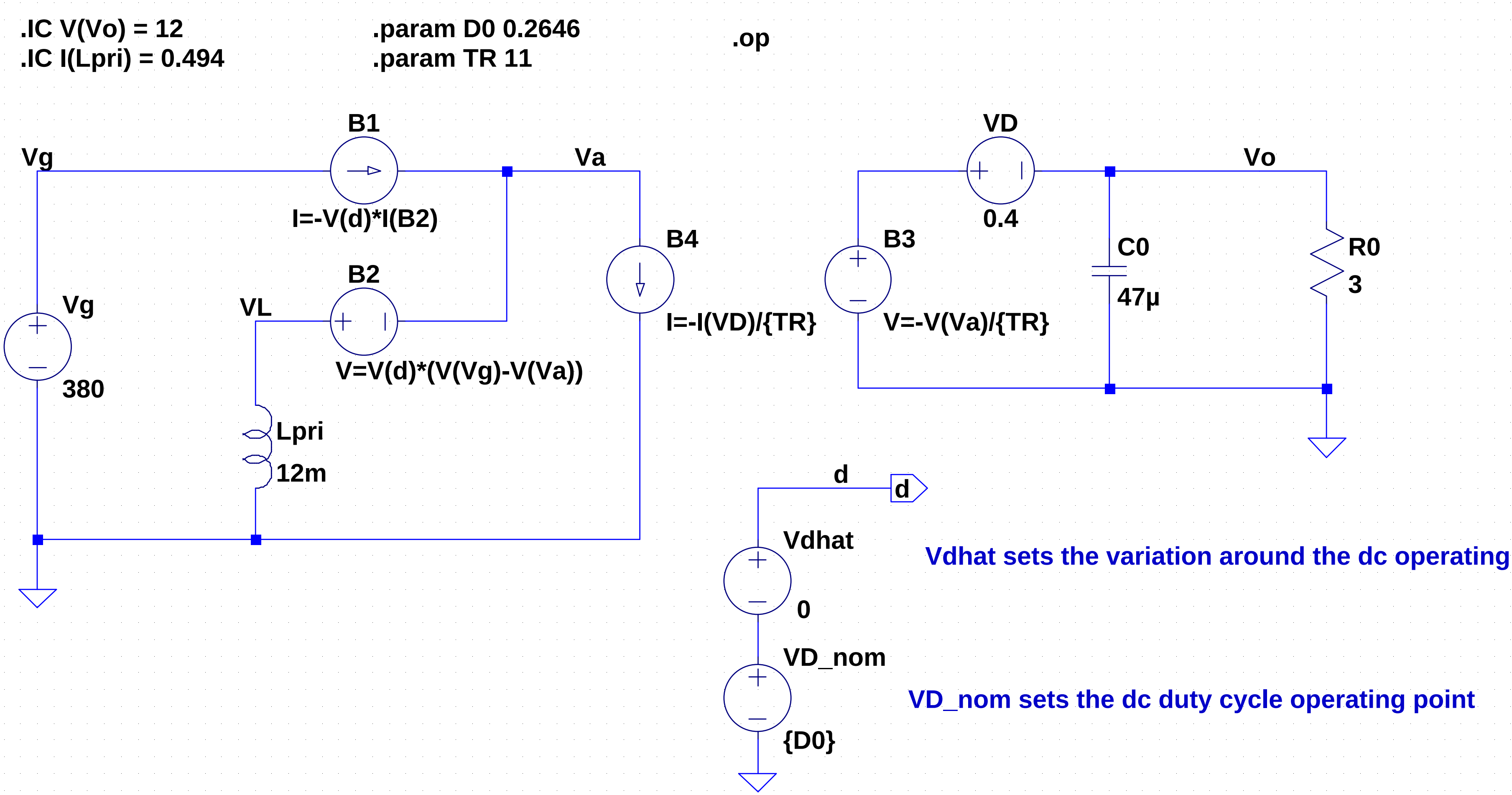
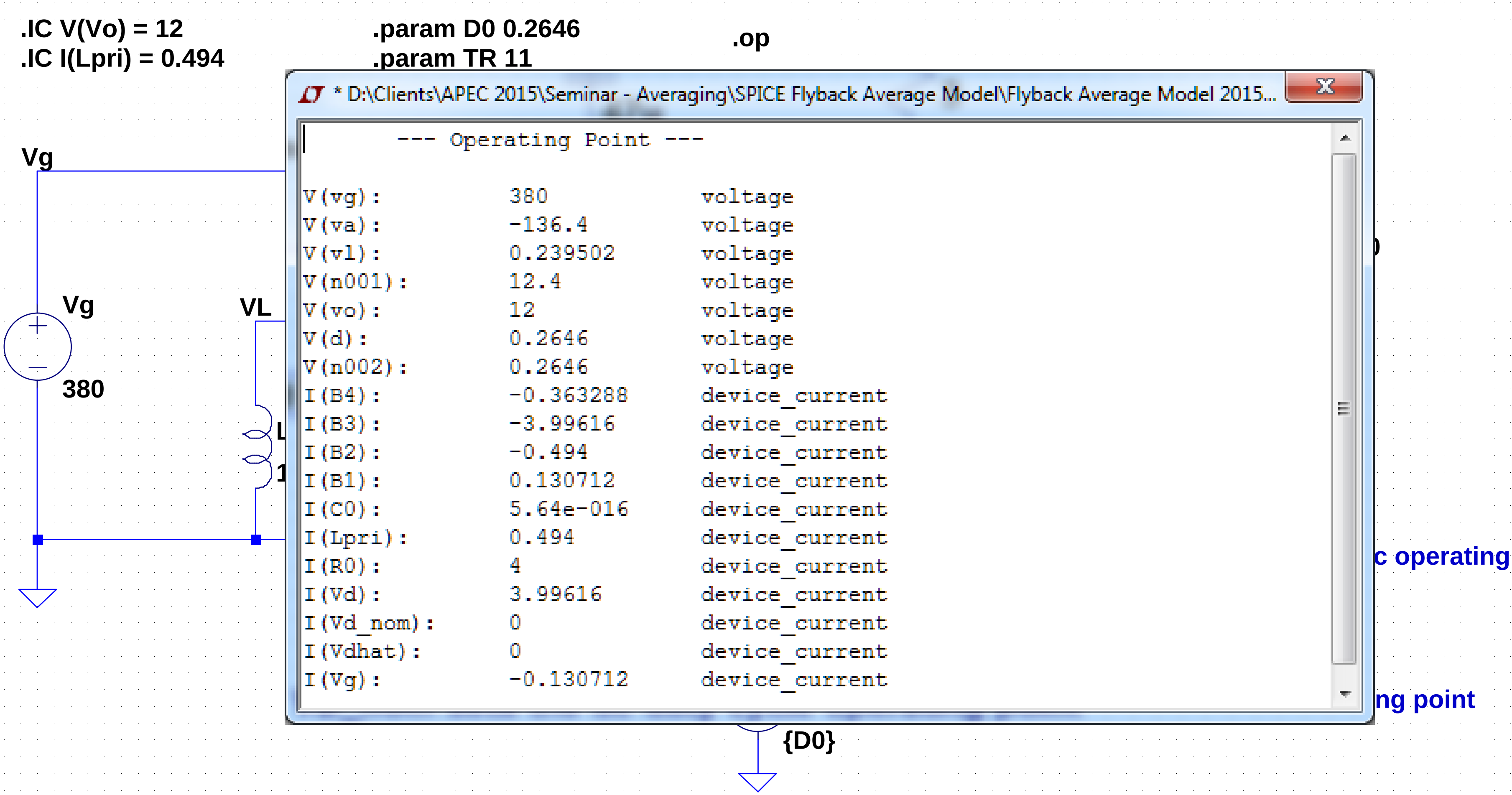
Flyback Simulation With Average Switch Model



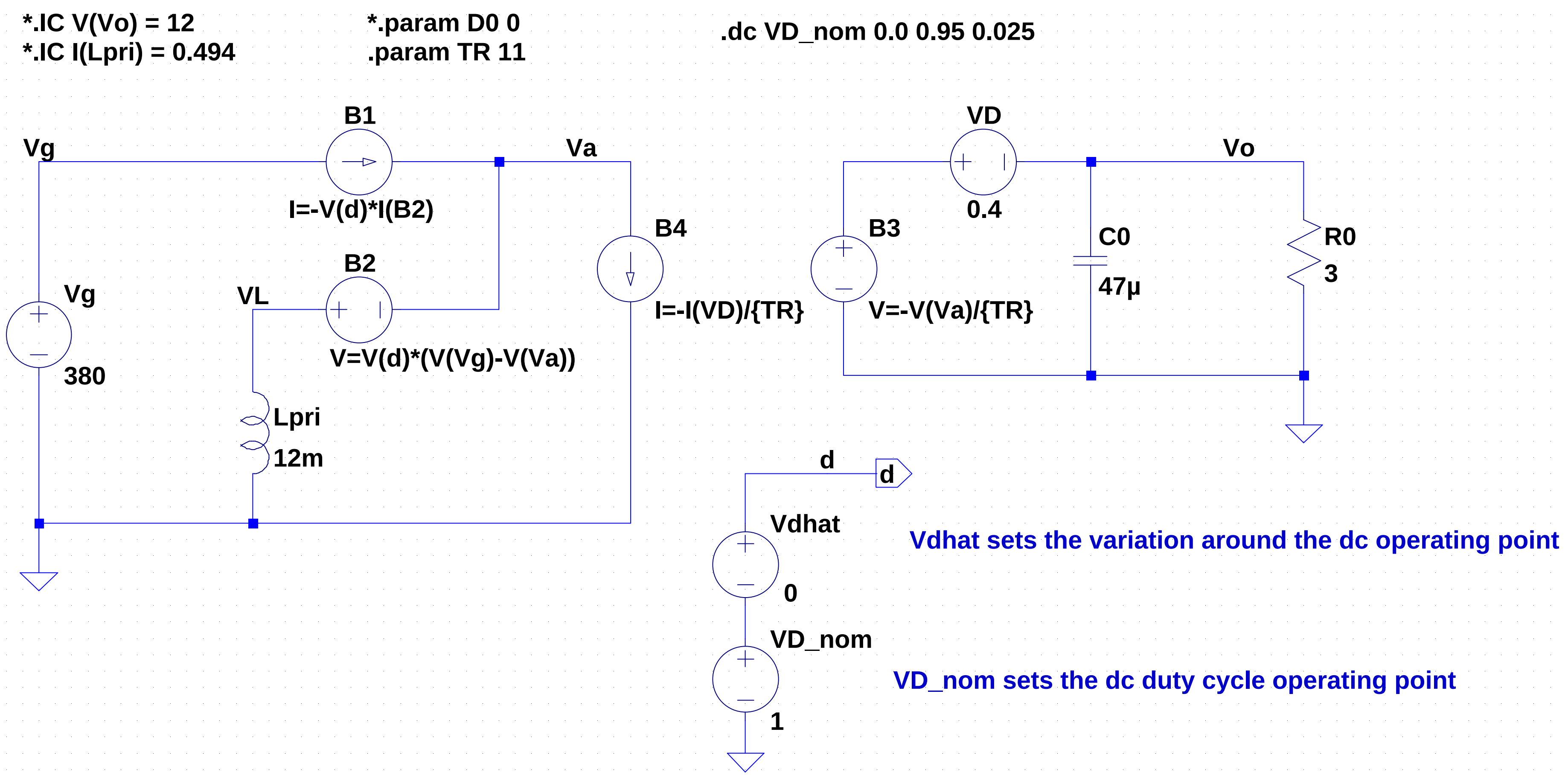
EMBEDDED POWER LABS

Flyback Simulation With Average Switch Model



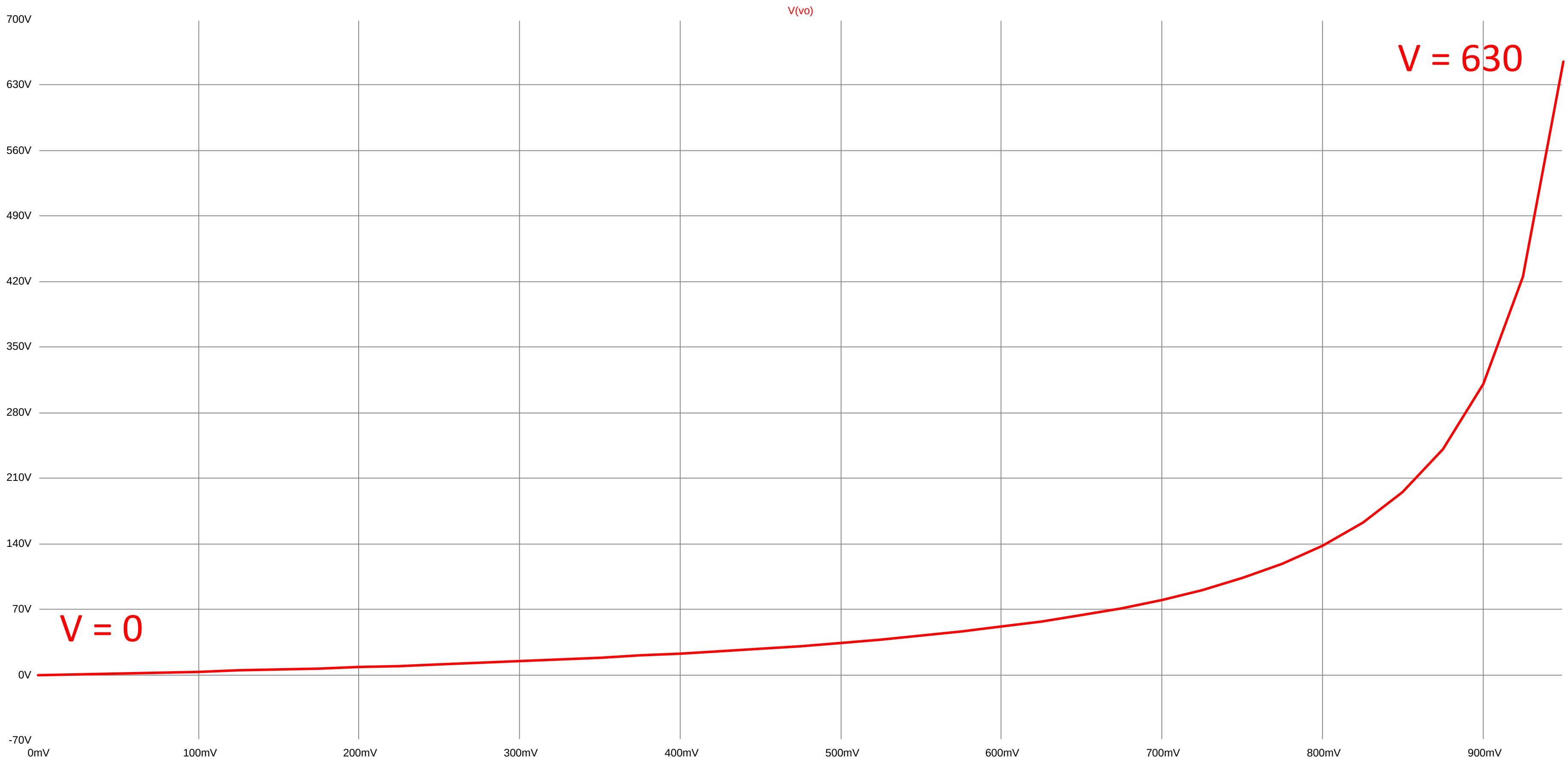
V(vg):	380	voltage
V(va):	-136.4	voltage
V(vl):	0.239502	voltage
V(n001):	12.4	voltage
V(vo):	12	voltage
V(d):	0.2646	voltage
V(n002):	0.2646	voltage
I(B4):	-0.363288	device_current
I(B3):	-3.99616	device_current
I(B2):	-0.494	device_current
I(B1):	0.130712	device_current
I(C0):	5.64e-016	device_current
I(Lpri):	0.494	device_current
I(R0):	4	device_current
I(Vd):	3.99616	device_current
I(Vd_nom):	0	device_current
I(Vdhat):	0	device_current
I(Vg):	-0.130712	device_current

Flyback Simulation With Average Switch Model



EMBEDDED POWER LABS

Flyback Simulation With Average Switch Model

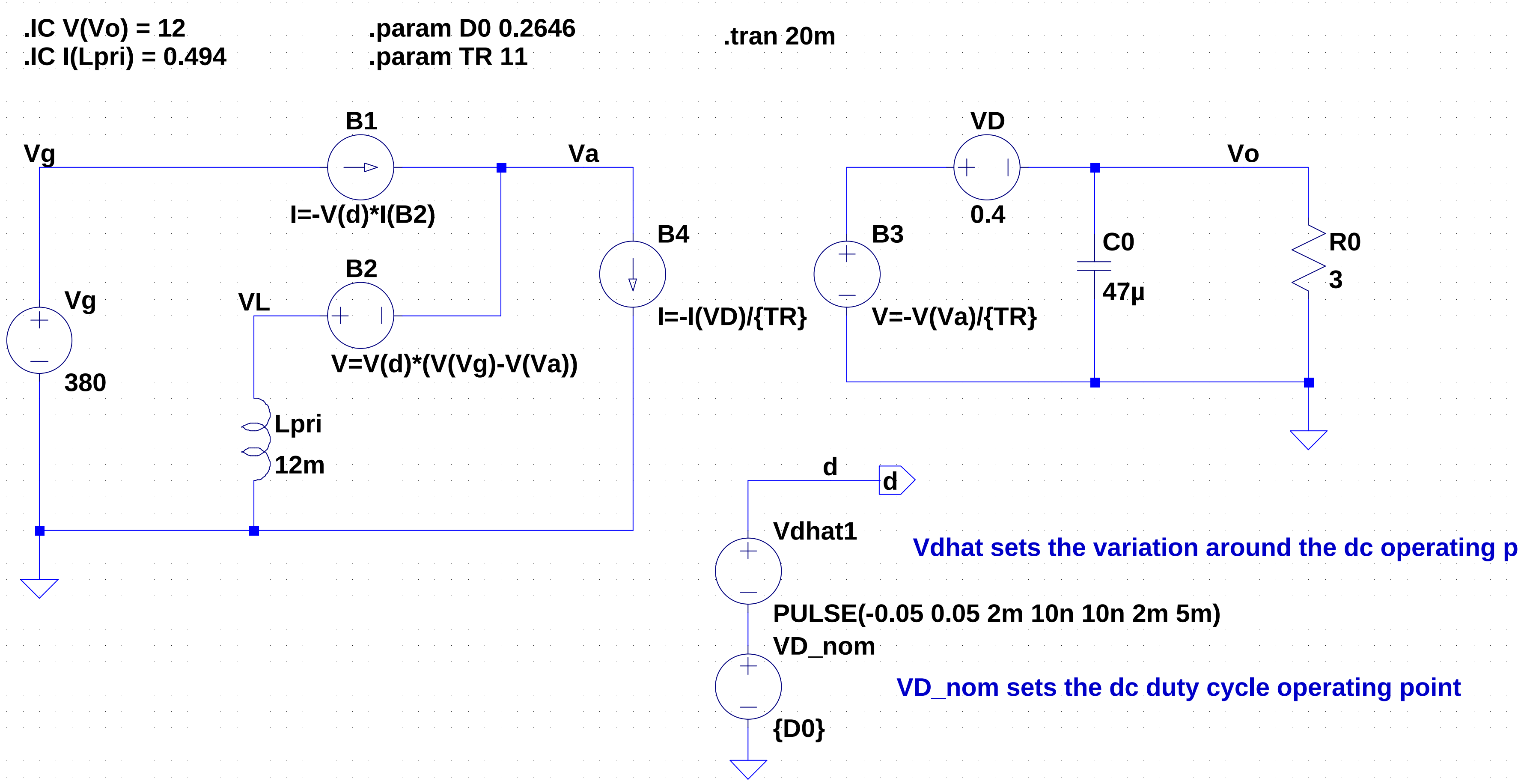


D =0

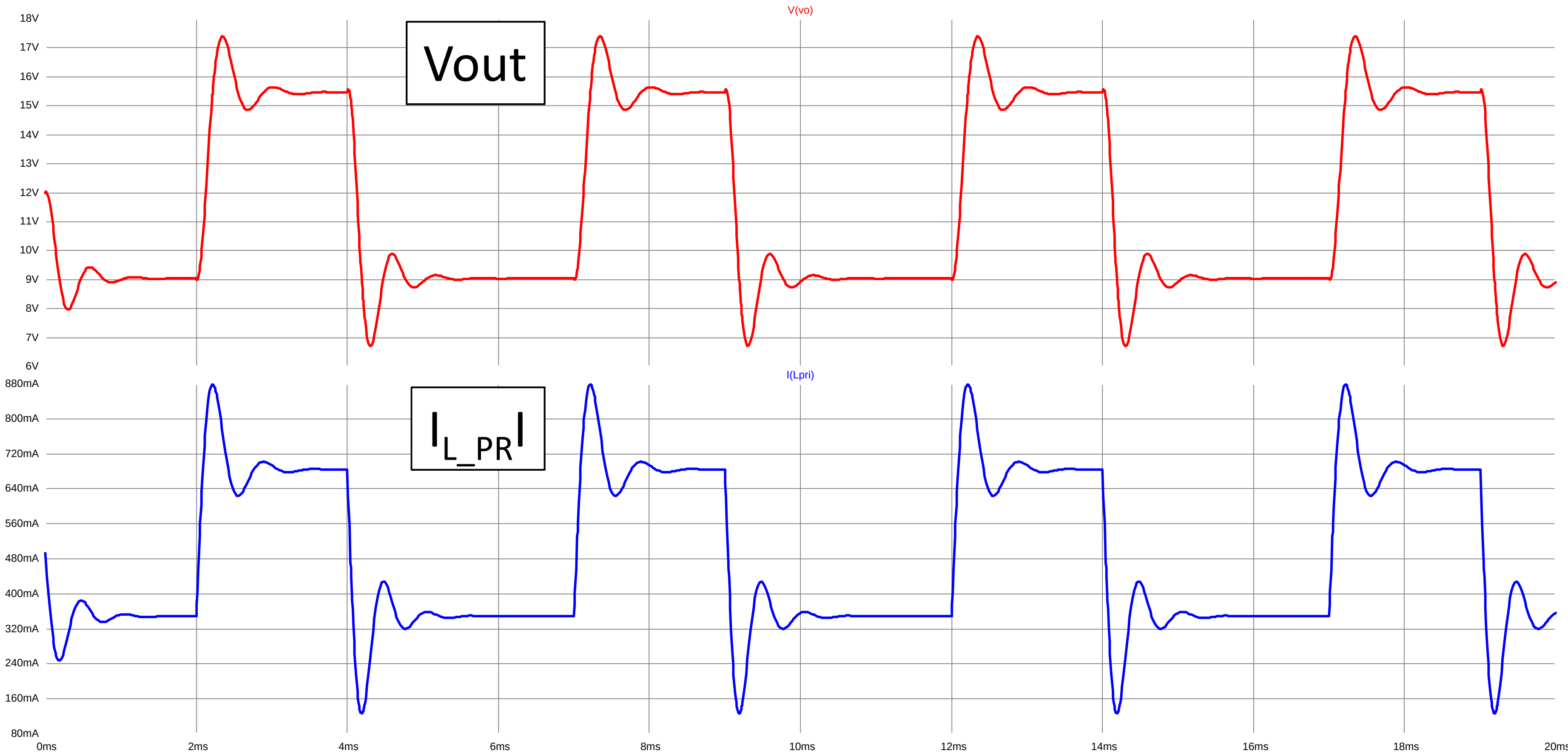
D =0.95

EMBEDDED POWER LABS

Flyback Simulation With Average Switch Model

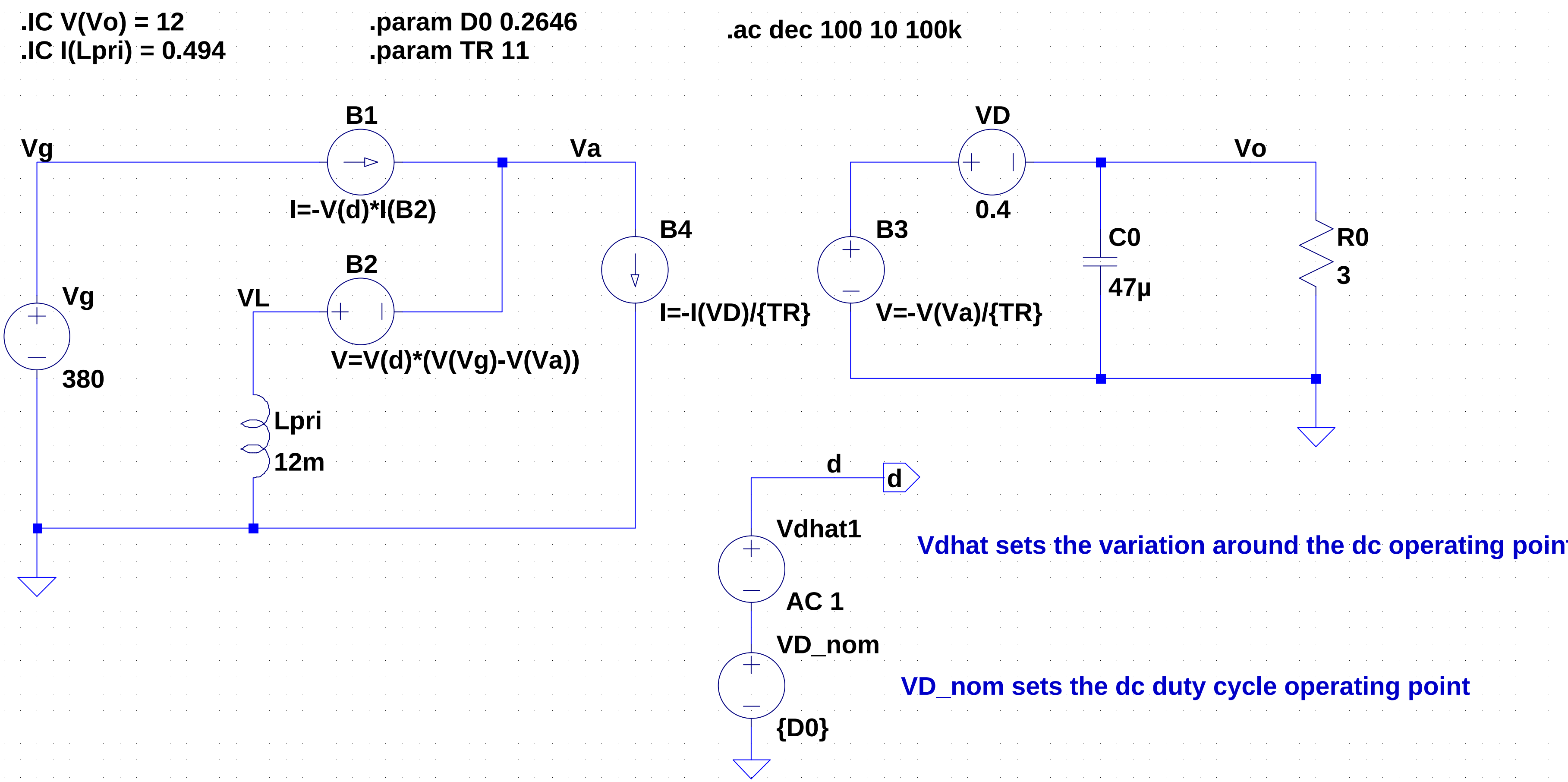


Flyback Simulation With Average Switch Model

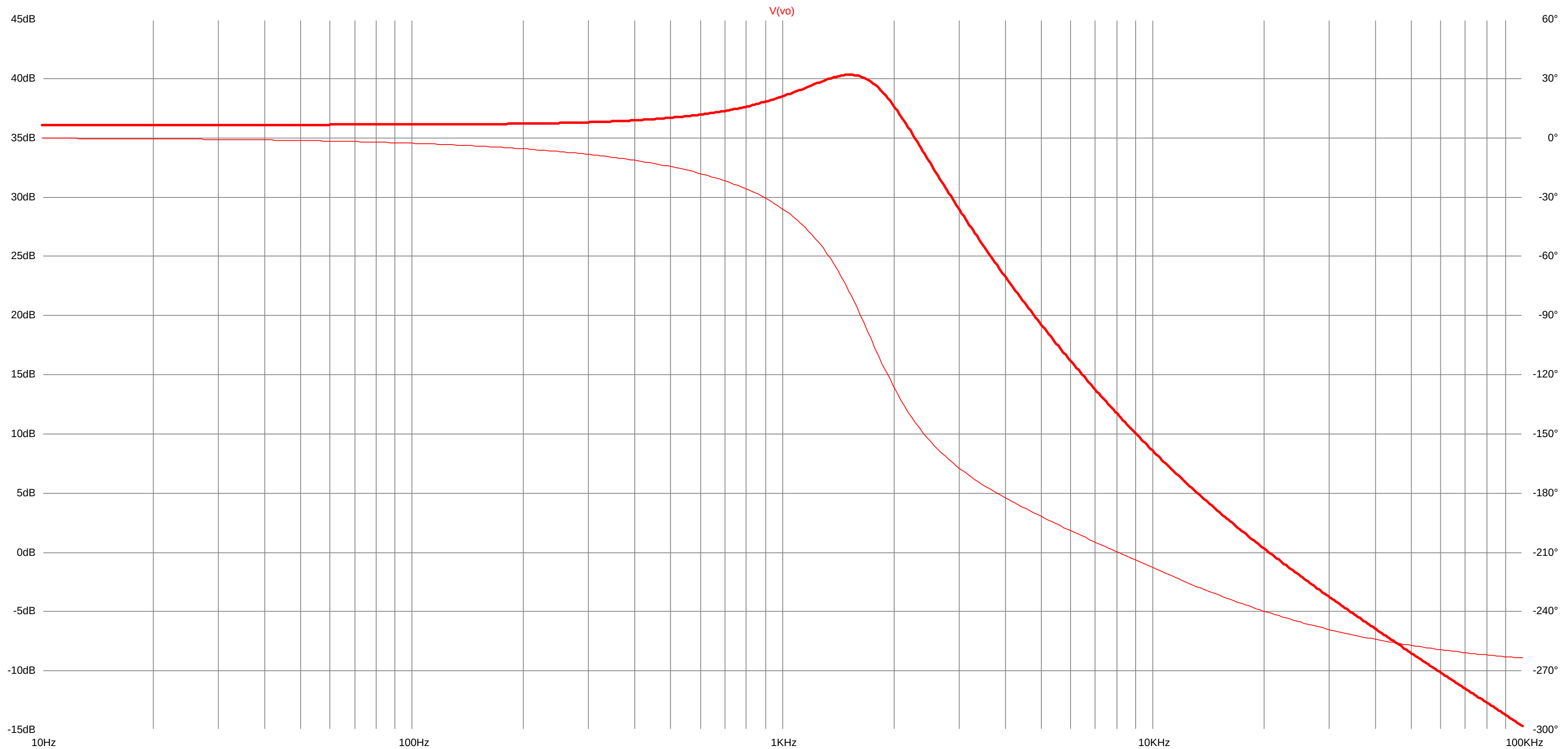


EMBEDDED POWER LABS

Flyback Simulation With Average Switch Model

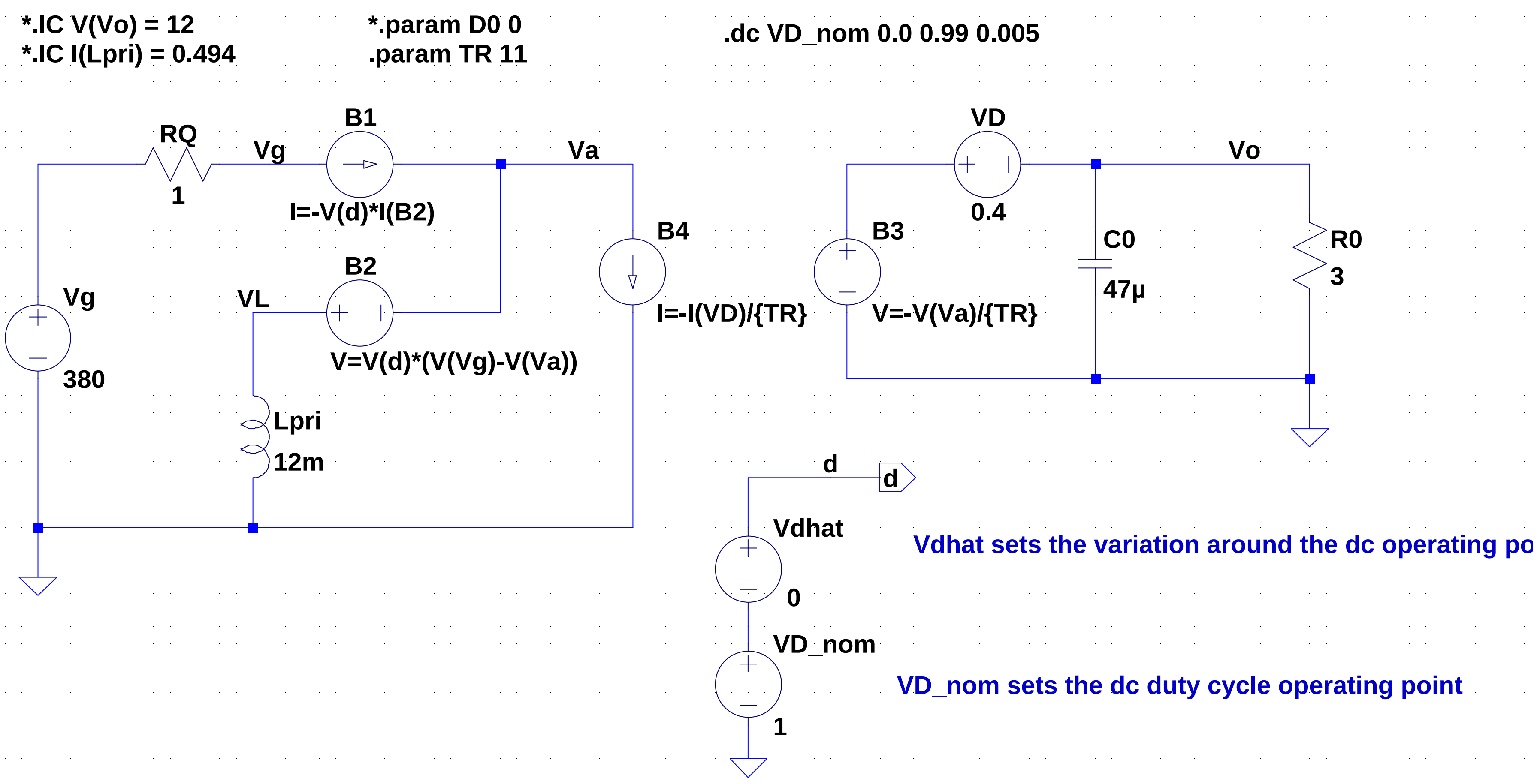


Flyback Simulation With Average Switch Model



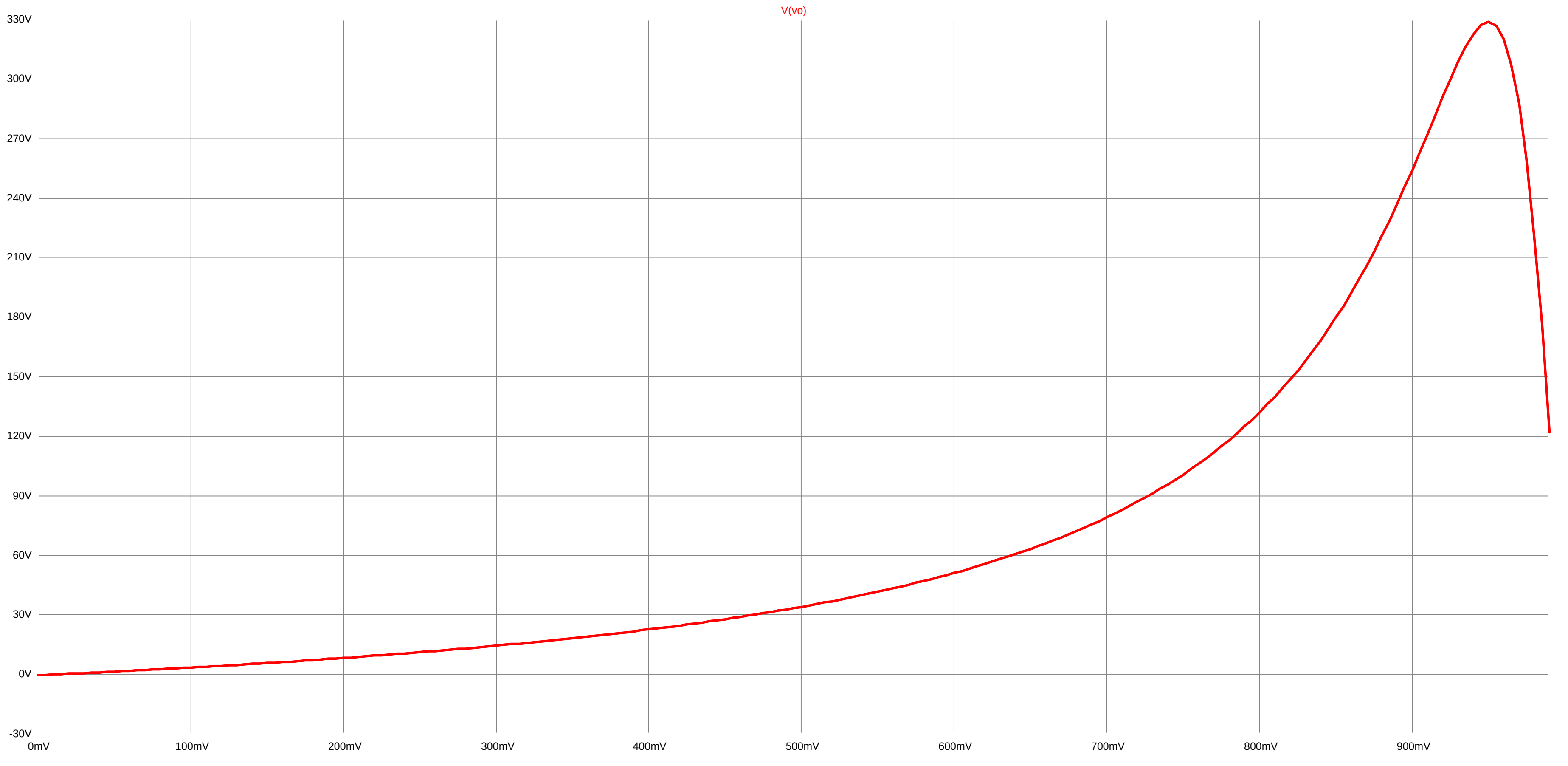
EMBEDDED POWER LABS

Flyback Simulation With Average Switch Model



EMBEDDED POWER LABS

Flyback Simulation With Average Switch Model



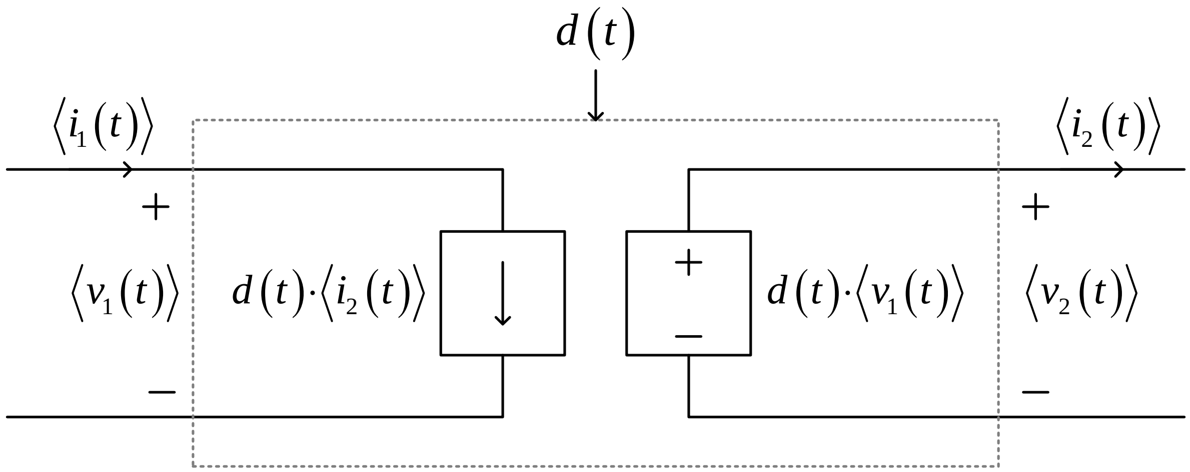
D =0

D =0.99

Embedded Power Labs

Averaged Switch Small Signal Model

!



$$\langle v_2(t)\rangle = d(t)\cdot\langle v_1(t)\rangle$$
$$\langle i_1(t)\rangle = d(t)\cdot\langle i_2(t)\rangle$$

$$\langle v_1(t)\rangle = V_1 + \hat{v}_1(t)$$
$$\langle v_2(t)\rangle = V_2 + \hat{v}_2(t)$$
$$\langle i_1(t)\rangle = I_1 + \hat{i}_1(t)$$
$$\langle i_2(t)\rangle = I_2 + \hat{i}_2(t)$$
$$d(t) = D + \hat{d}(t)$$

Averaged Switch Small Signal Model

$$\langle v_2(t)\rangle = d(t)\cdot\langle v_1(t)\rangle$$

$$V_2 + \hat{v}_2(t) = \left(D + \hat{d}(t)\right)\cdot\left(V_1 + \hat{v}_1(t)\right)$$

$$= D\cdot V_1 + D\cdot\hat{v}_1(t) + V_1\cdot\hat{d}(t) + \hat{d}(t)\cdot\hat{v}_1(t)$$

$$\hat{d}(t)\cdot\hat{v}_1(t) \approx 0$$

Discard Higher Order Nonlinear Terms

$$V_2 + \hat{v}_2(t) = D\cdot V_1 + D\cdot\hat{v}_1(t) + V_1\cdot\hat{d}(t)$$

$$= D\cdot\left(V_1 + \hat{v}_1(t)\right) + V_1\cdot\hat{d}(t)$$

DC And Small Signal Terms!

Averaged Switch Small Signal Model

$$\langle i_1(t) \rangle = d(t) \cdot \langle i_2(t) \rangle$$

$$I_1 + \hat{i}_1(t) = \left(D + \hat{d}(t)\right) \cdot \left(I_2 + \hat{i}_2(t)\right)$$

$$= D \cdot I_2 + D \cdot \hat{i}_2(t) + I_2 \cdot \hat{d}(t) + \hat{d}(t) \cdot \hat{i}_2(t)$$

$$\hat{d}(t) \cdot \hat{i}_2(t) \approx 0$$

$$I_1 + \hat{i}_1(t) = D \cdot I_2 + D \cdot \hat{i}_2(t) + I_2 \cdot \hat{d}(t)$$

$$= D \cdot \left(I_2 + \hat{i}_2(t)\right) + I_2 \cdot \hat{d}(t)$$

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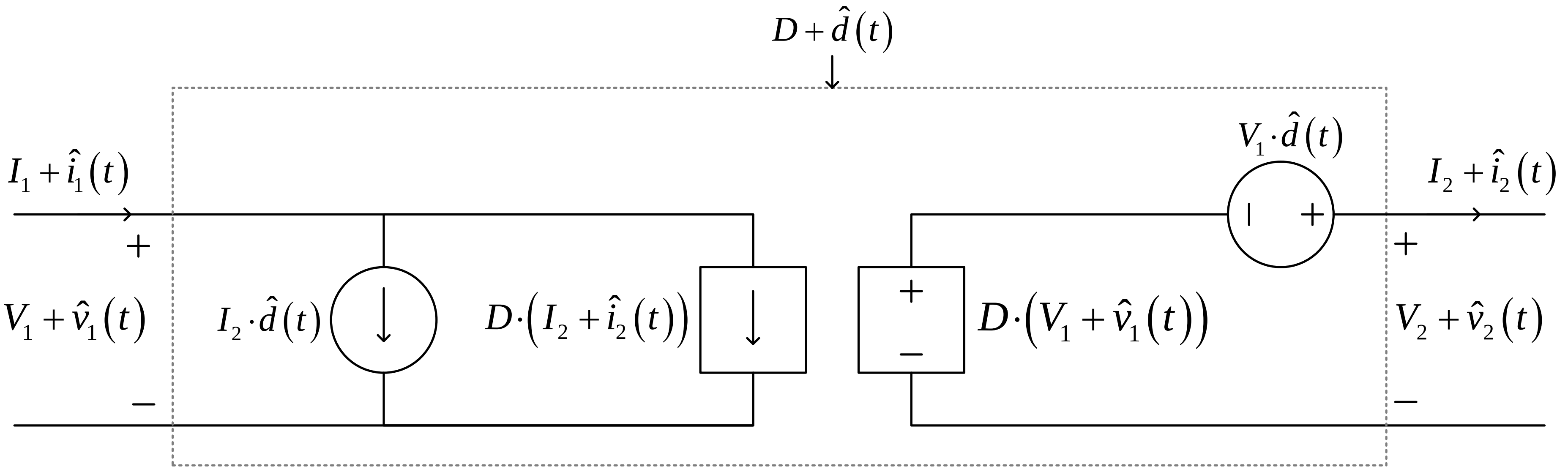
Construct The Switch Model

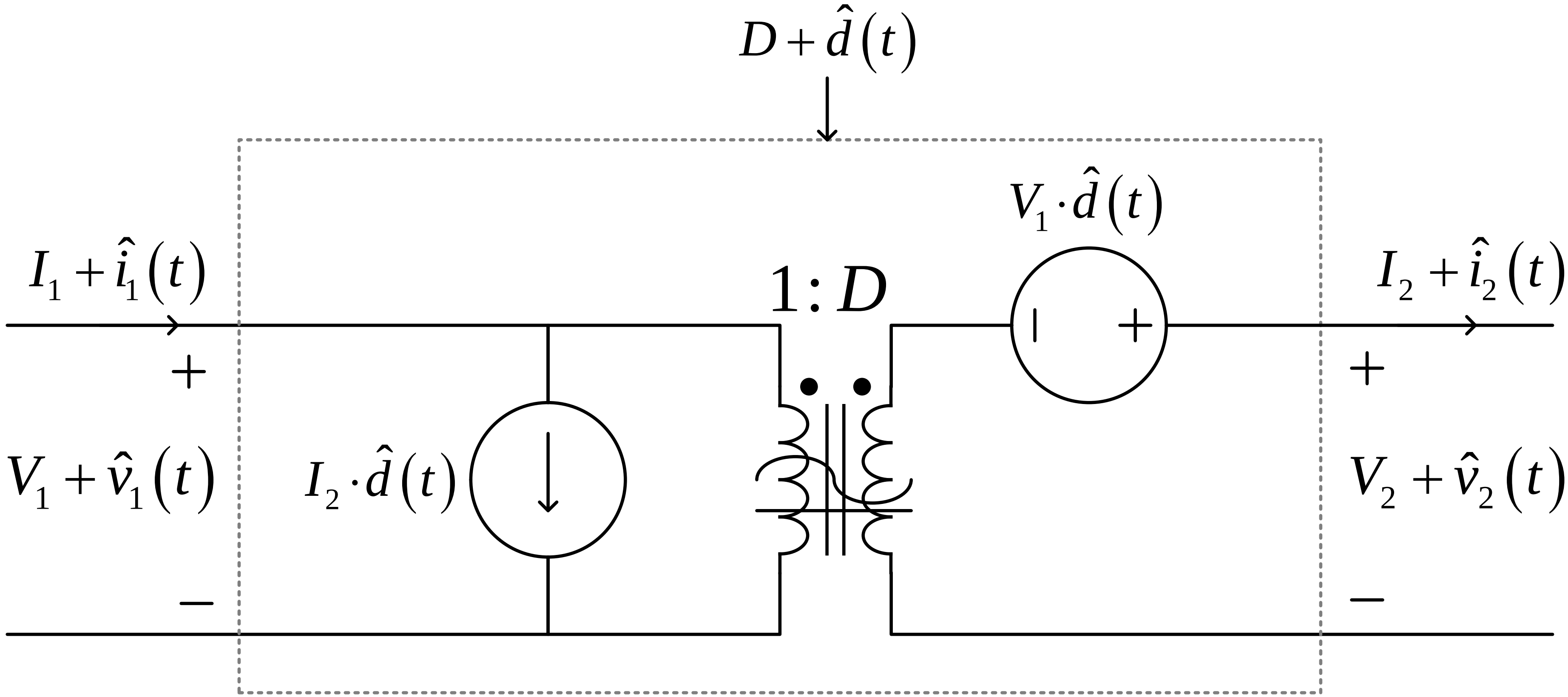
$$I_1 + \hat{i}_1(t) = D \cdot \left(I_2 + \hat{i}_2(t)\right) + I_2 \cdot \hat{d}(t) \qquad V_2 + \hat{v}_2(t) = D \cdot \left(V_1 + \hat{v}_1(t)\right) + V_1 \cdot \hat{d}(t)$$



EMBEDDED POWER LABS

Construct The Switch Model

$$I_1 + \hat{i}_1(t) = D \bullet \left(I_2 + \hat{i}_2(t)\right) + I_2 \bullet \hat{d}(t) \qquad V_2 + \hat{v}_2(t) = D \bullet \left(V_1 + \hat{v}_1(t)\right) + V_1 \bullet \hat{d}(t)$$



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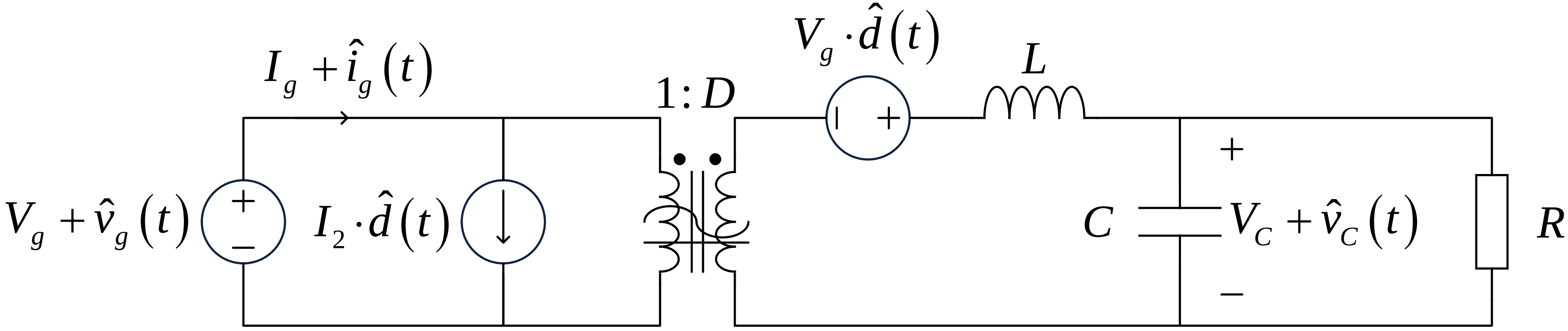
Averaged Switch Small Signal Buck Converter Model



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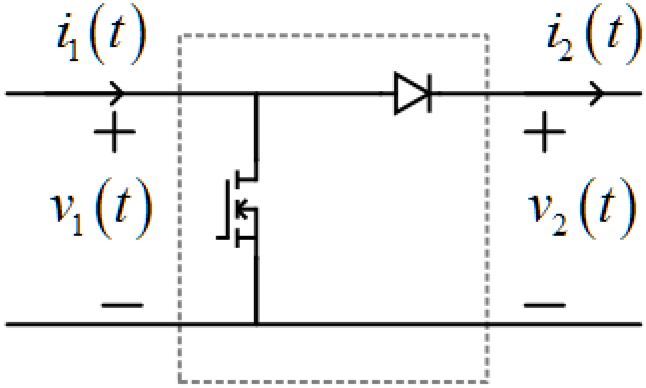
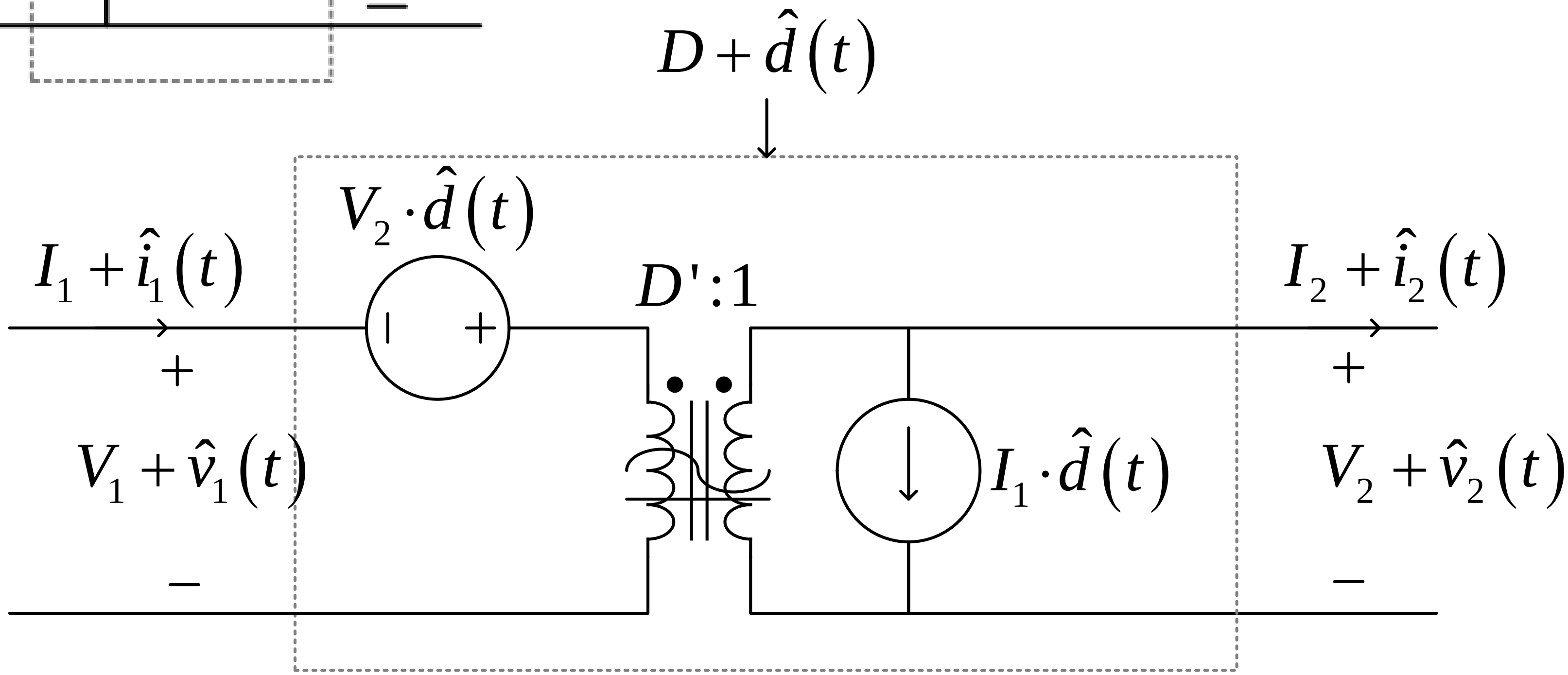
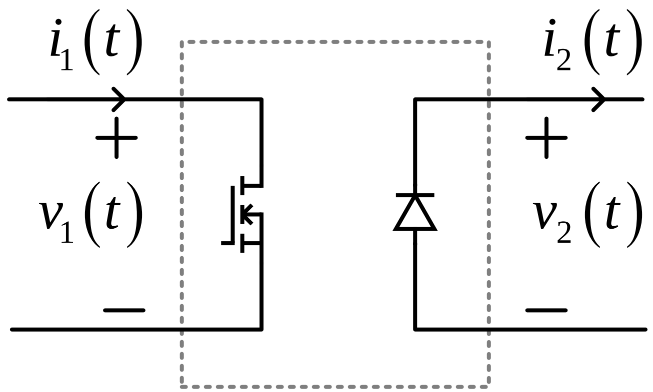
Boost Switch Model

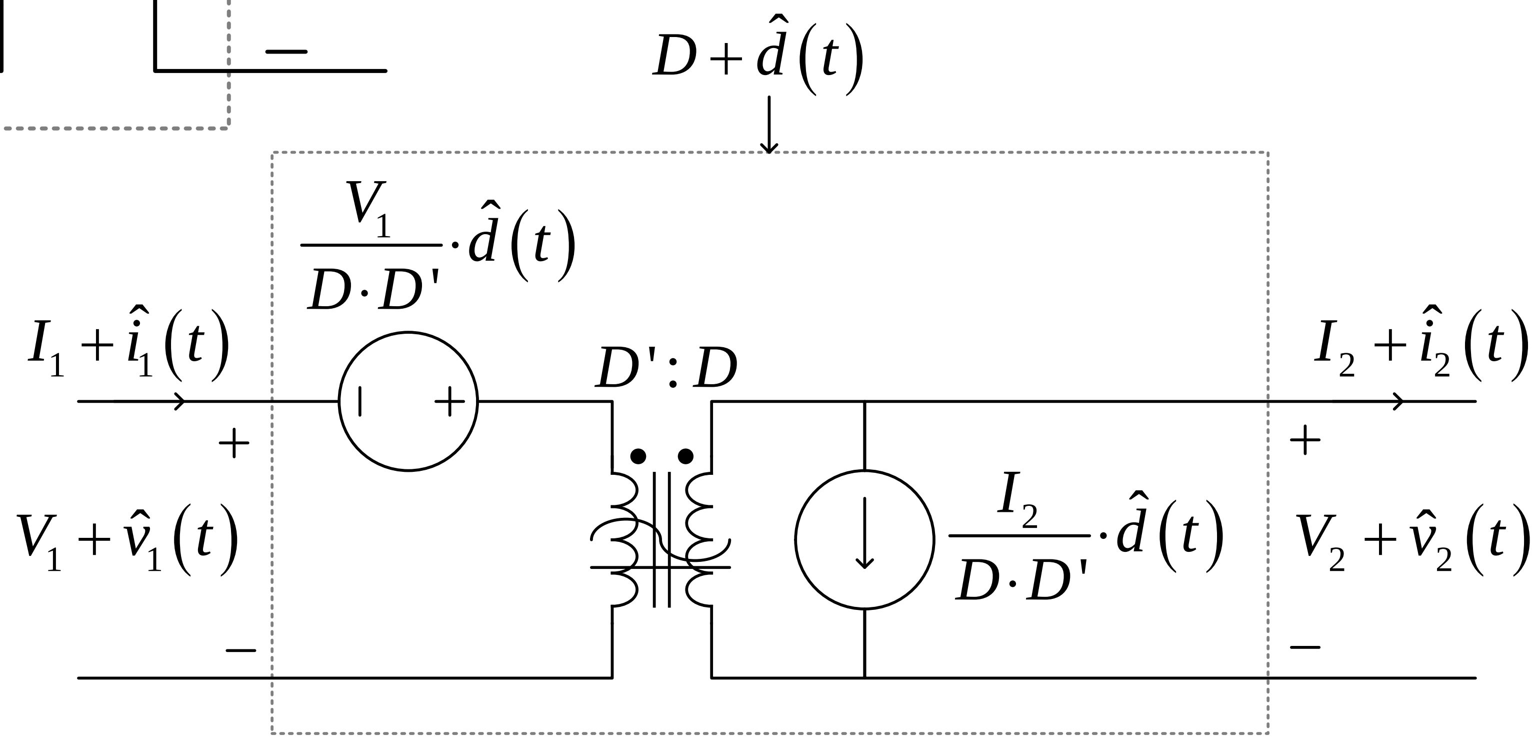




Re-drawn from "Fundamentals of Power Electronics", 2nd ed., Erickson and Maksimovic, Figure 7.50

General Two-Switch Network





Re-drawn from "Fundamentals of Power Electronics", 2nd ed., Erickson and Maksimovic, Figure 7.50

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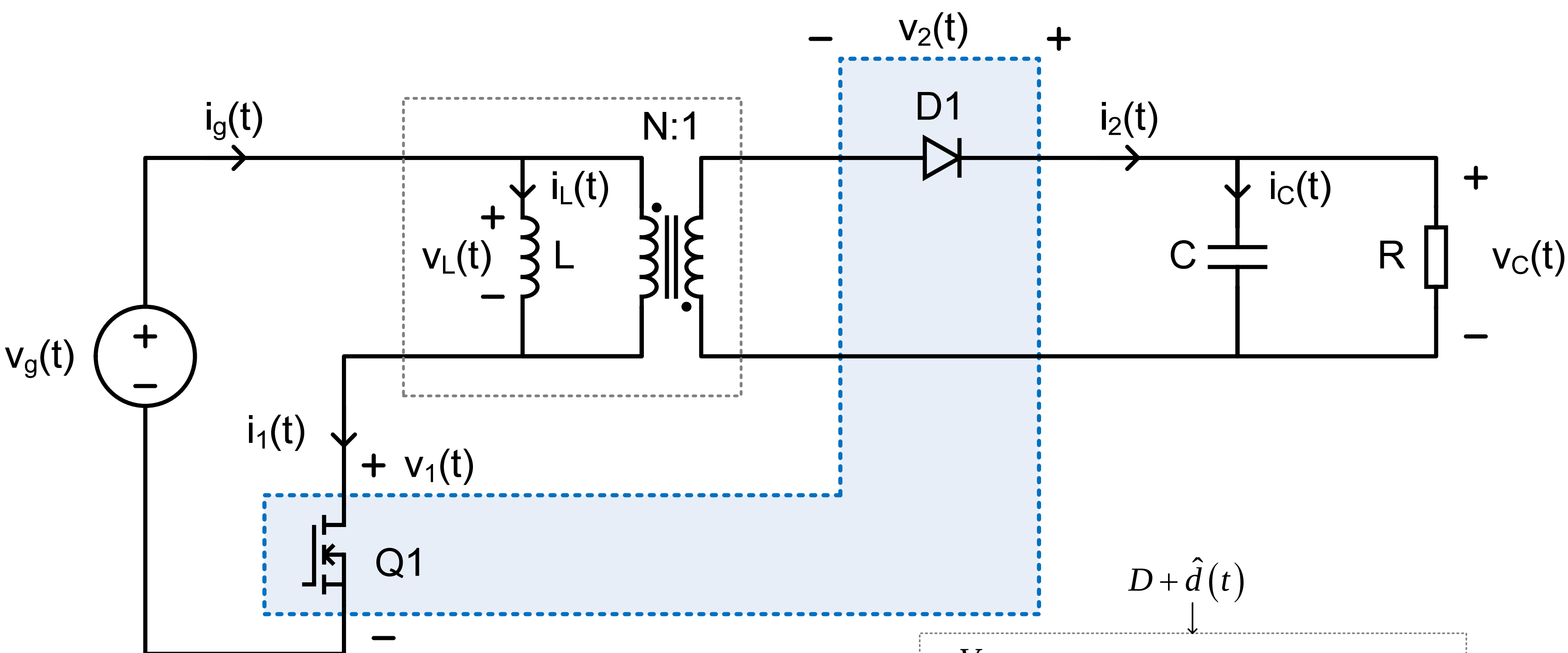
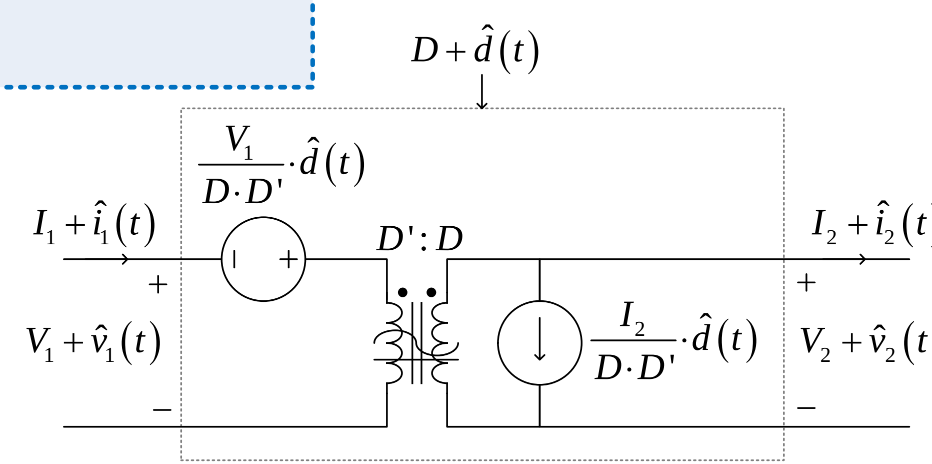
Averaged Switch Small Signal Model: Flyback





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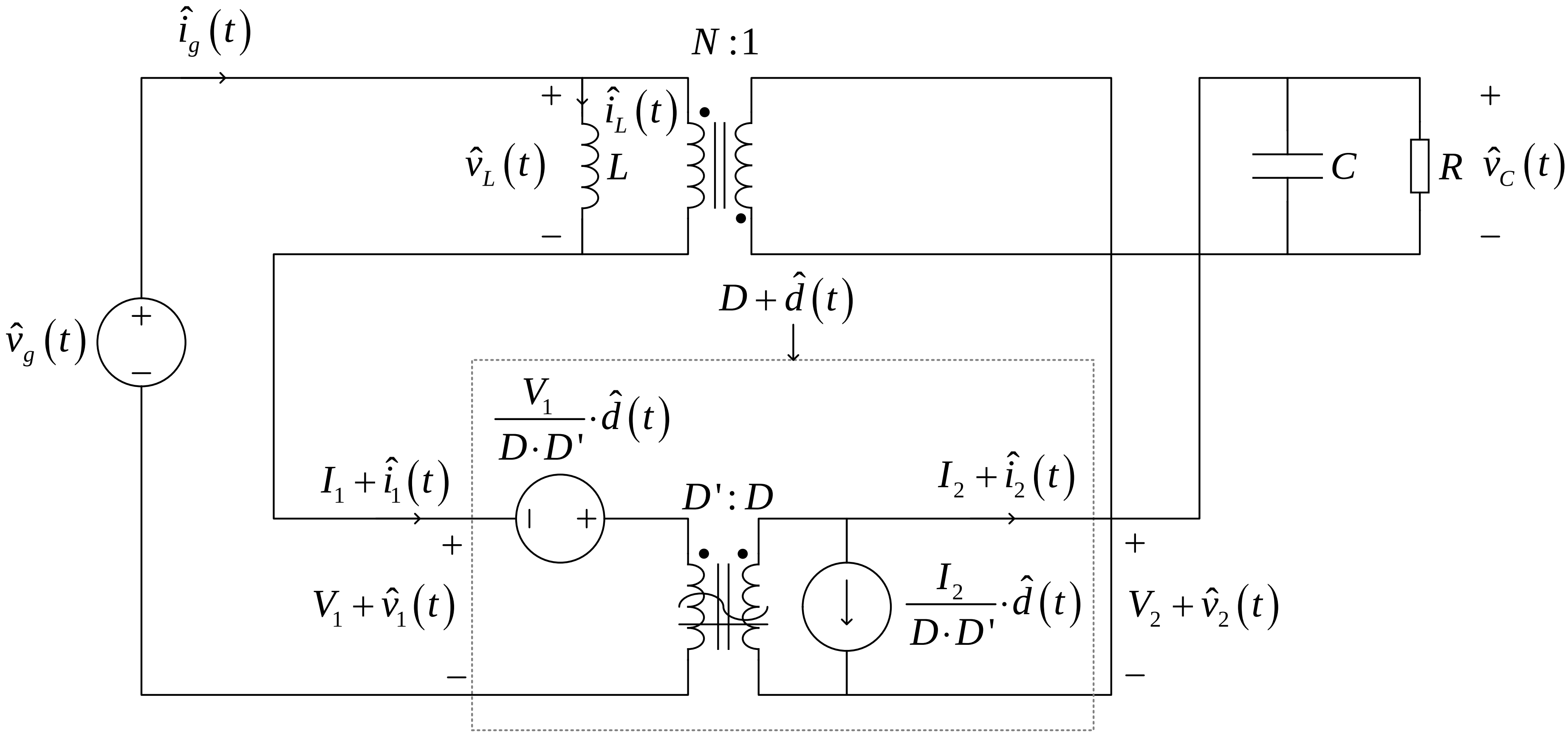
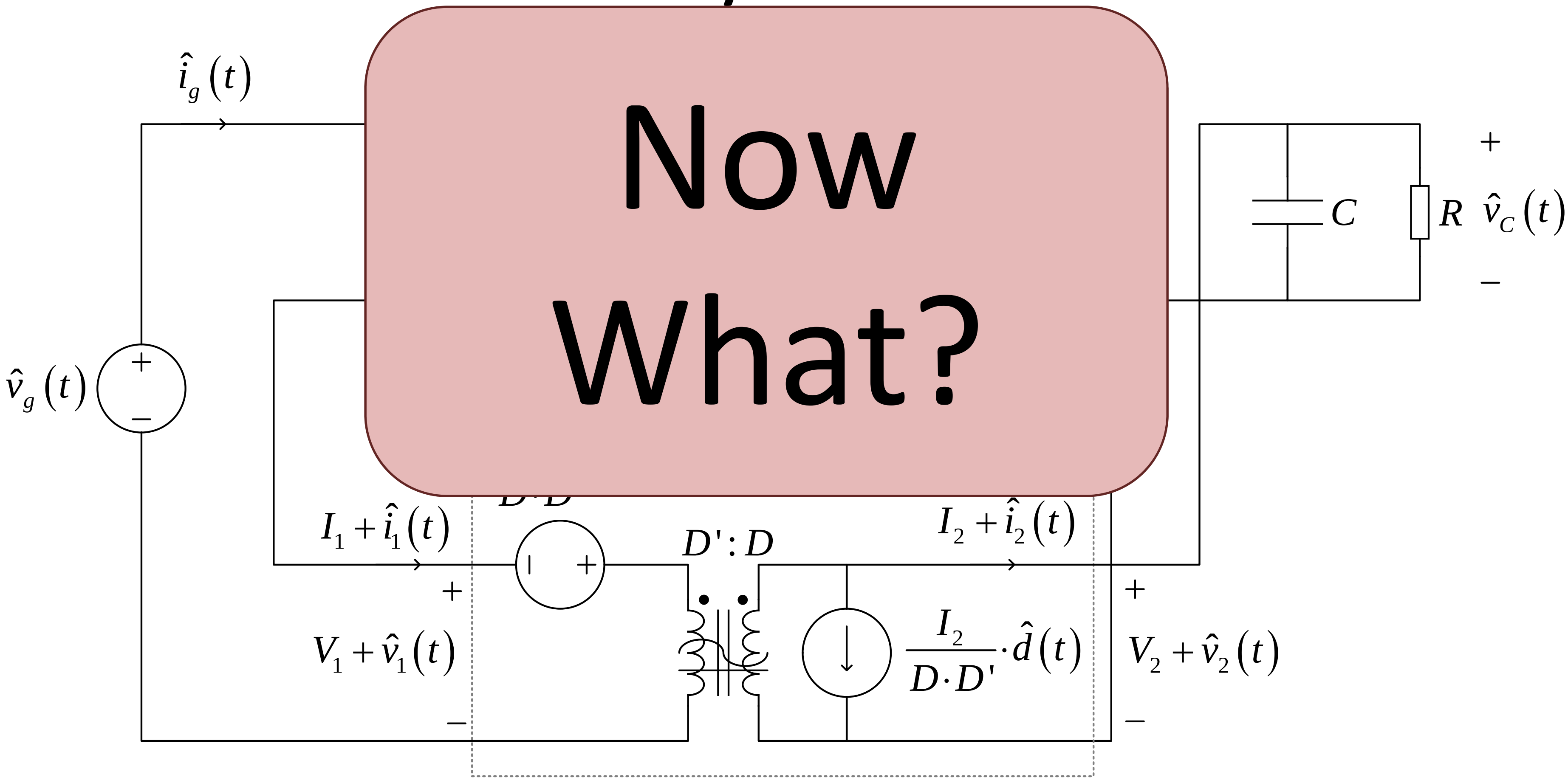
Averaged Switch Small Signal Model: Flyback



Averaged Switch Small Signal Model: Flyback



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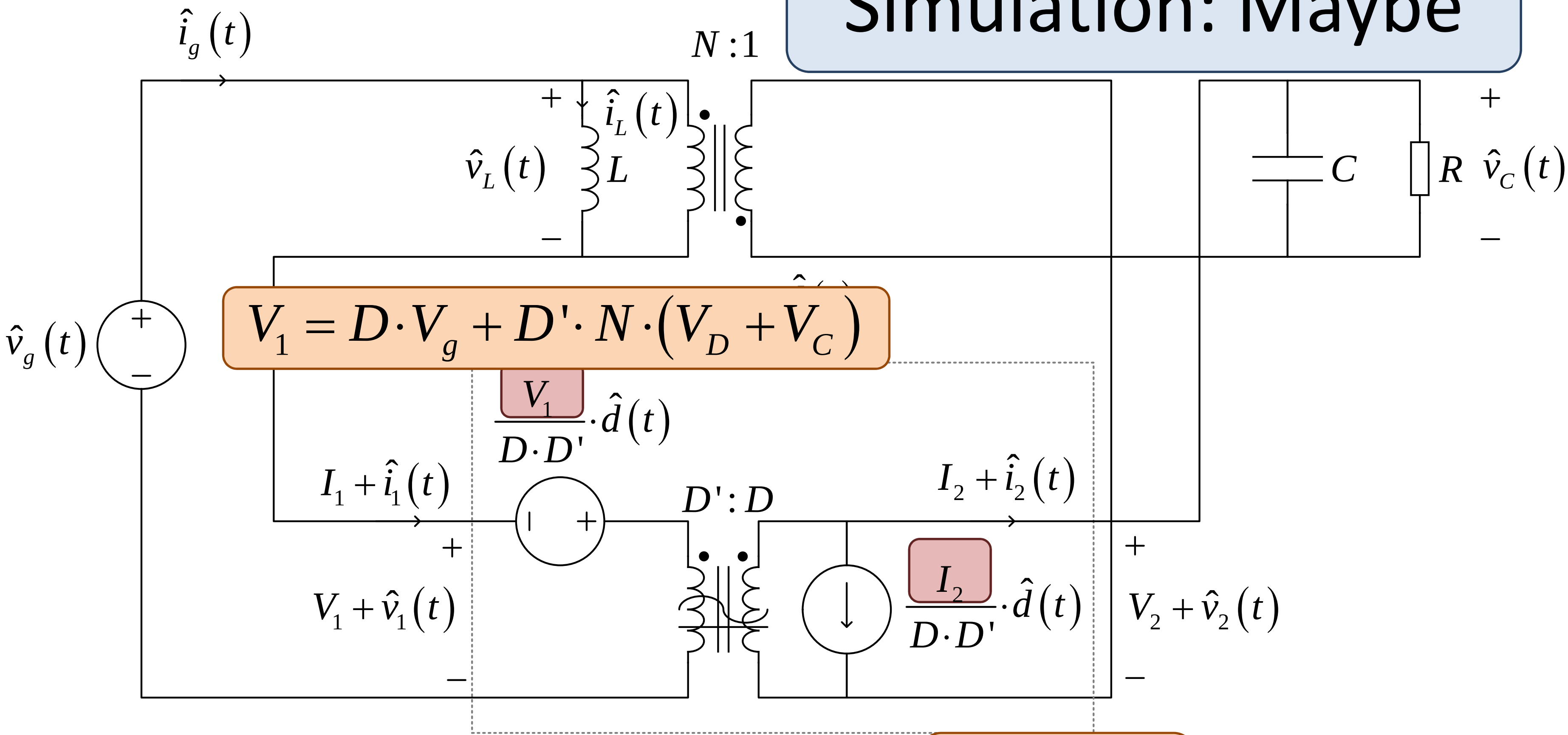
Averaged Switch Small Signal Model: Flyback

Simulation: Maybe



$$V_1 = D \cdot V_g + D' \cdot N \cdot (V_D + V_C)$$

$$I_2 = \frac{1}{R} \cdot V_C$$

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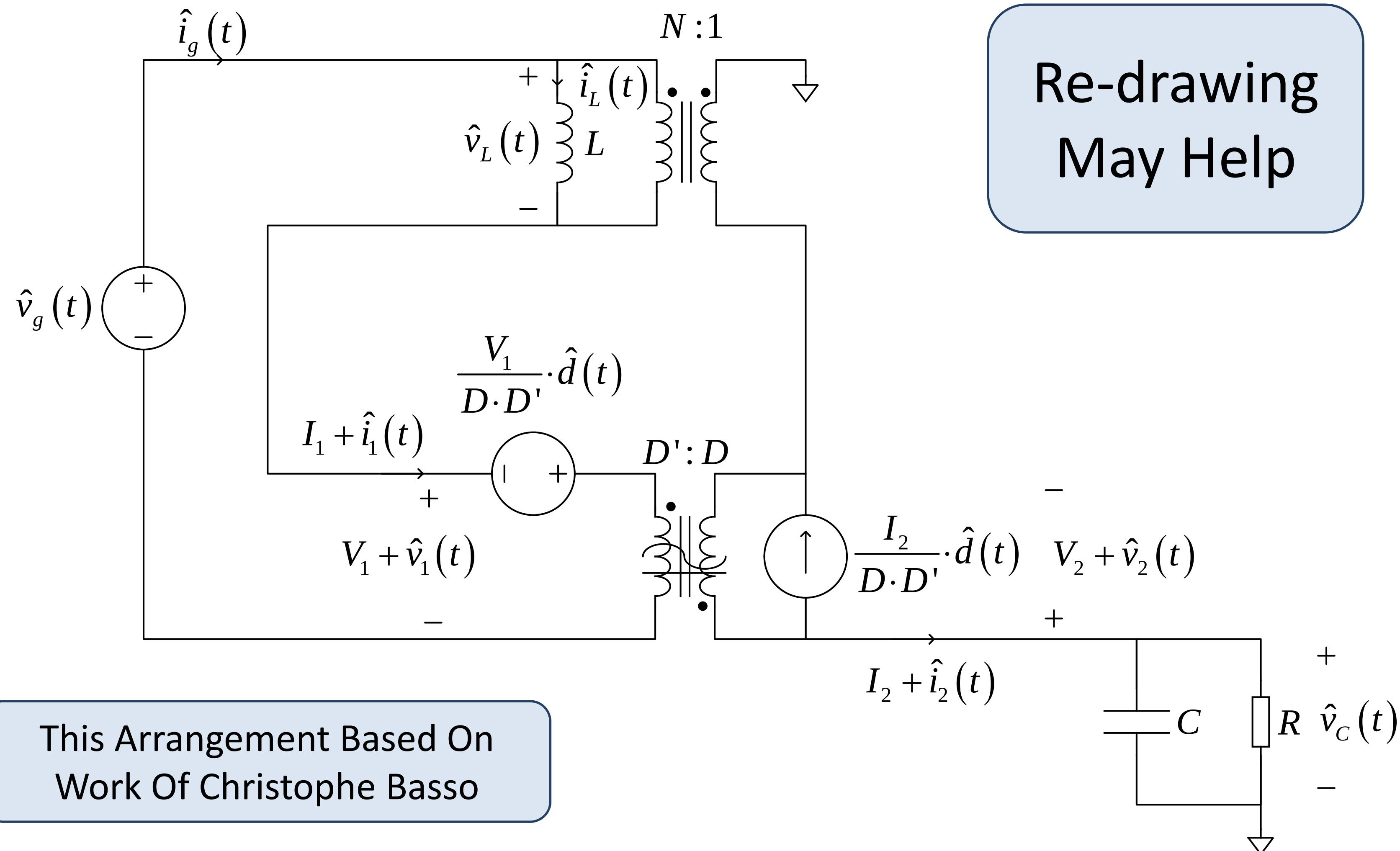
Averaged Switch Small Signal Model: Flyback



Re-drawing May Help

This Arrangement Based On Work Of Christophe Basso

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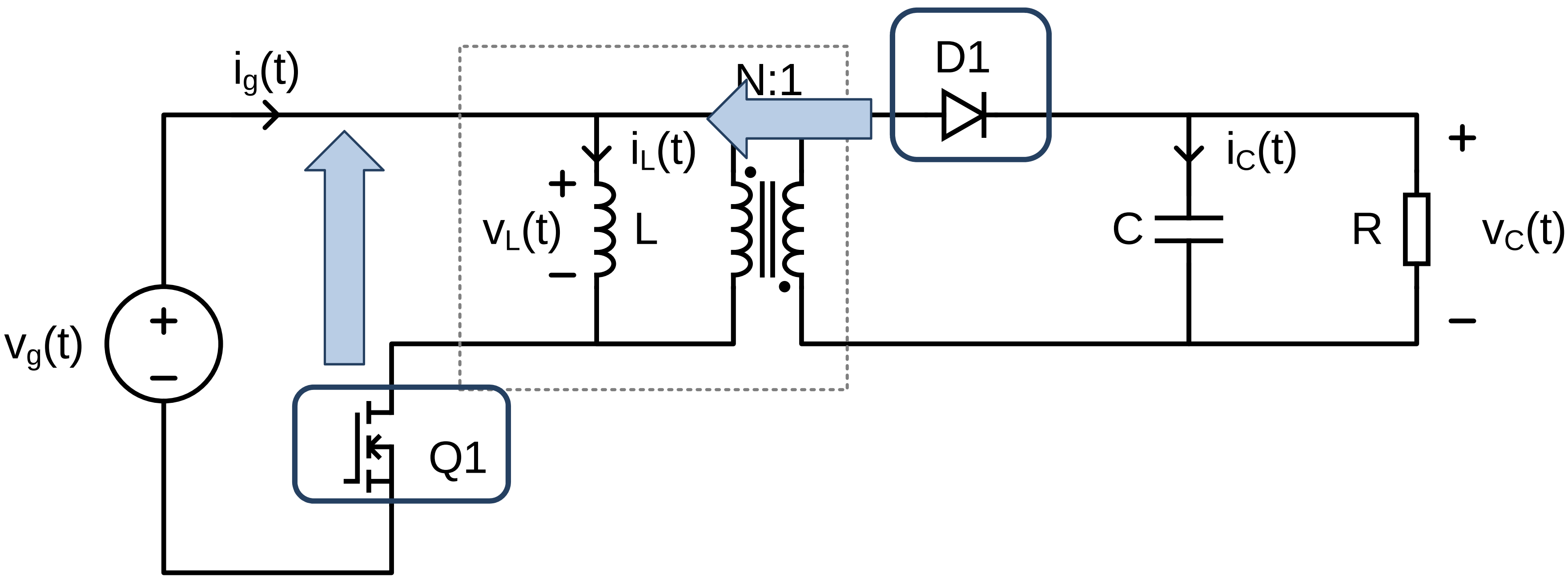
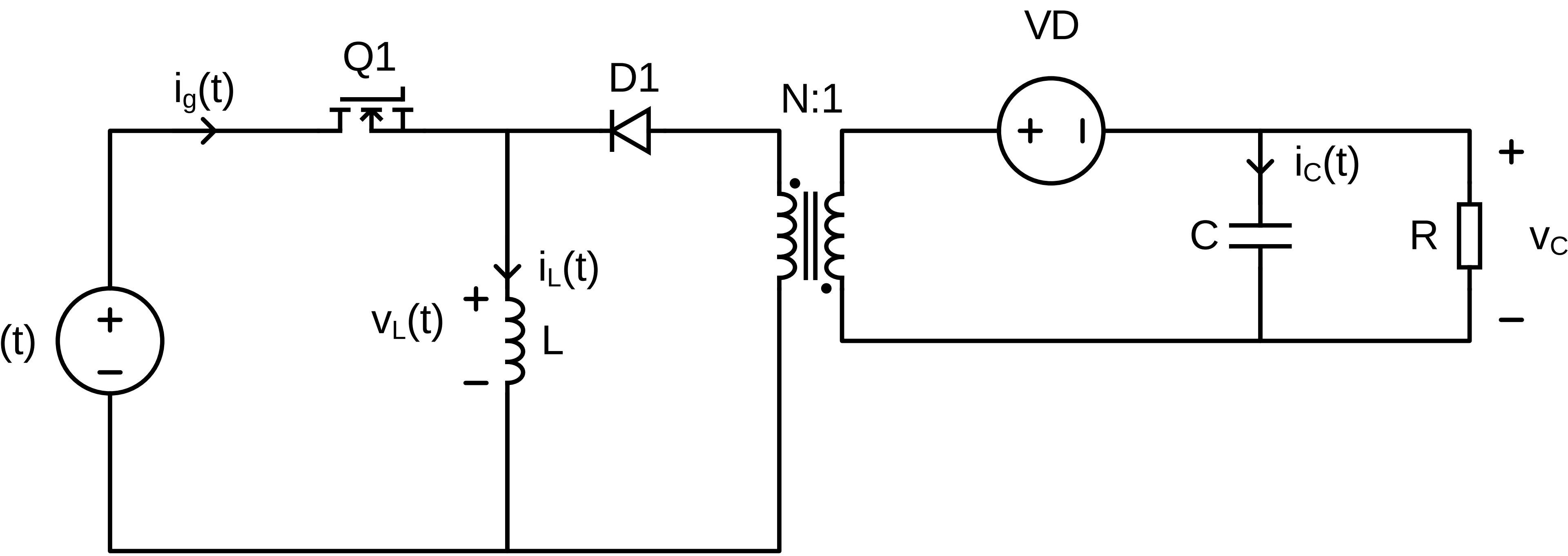
Averaged Switch Small Signal Model: Flyback

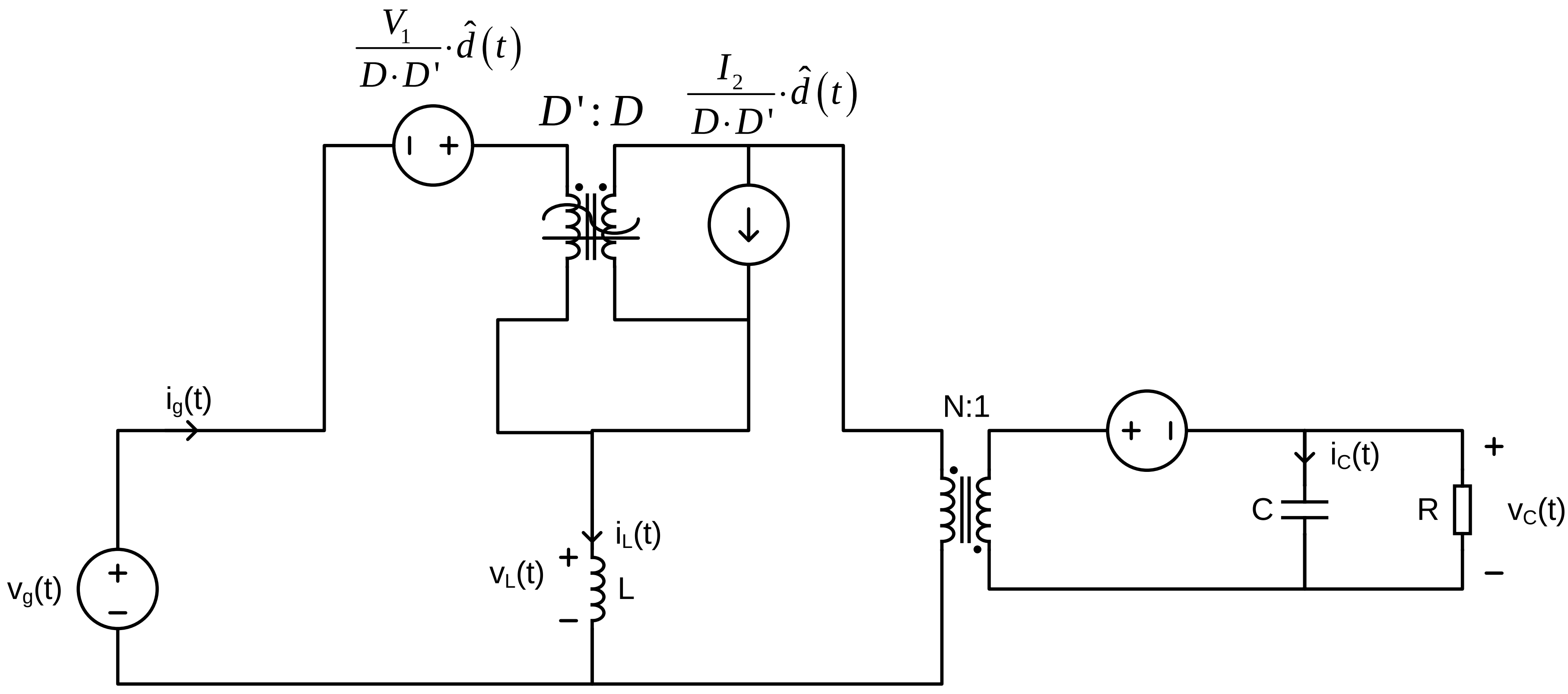
Re-draw
As For Averaged
Model



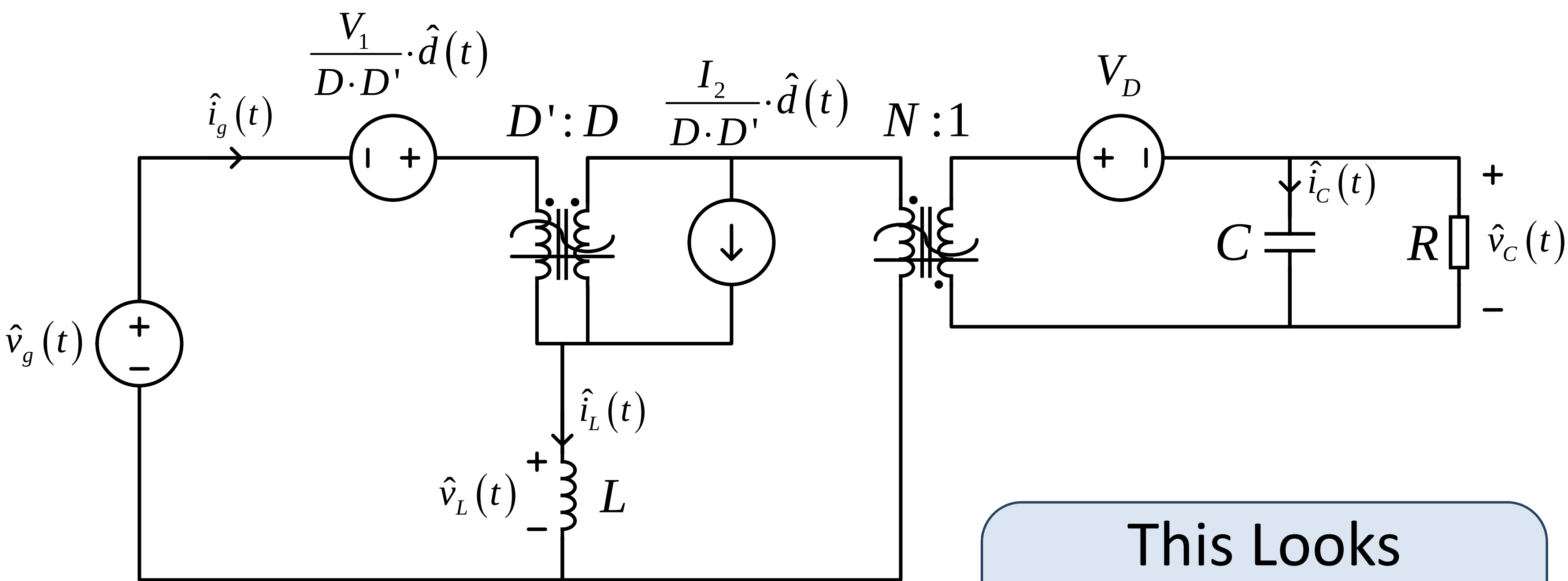
Averaged Switch Small Signal Model: Flyback



Averaged Switch Small Signal Model: Flyback



Averaged Switch Small Signal Model: Flyback

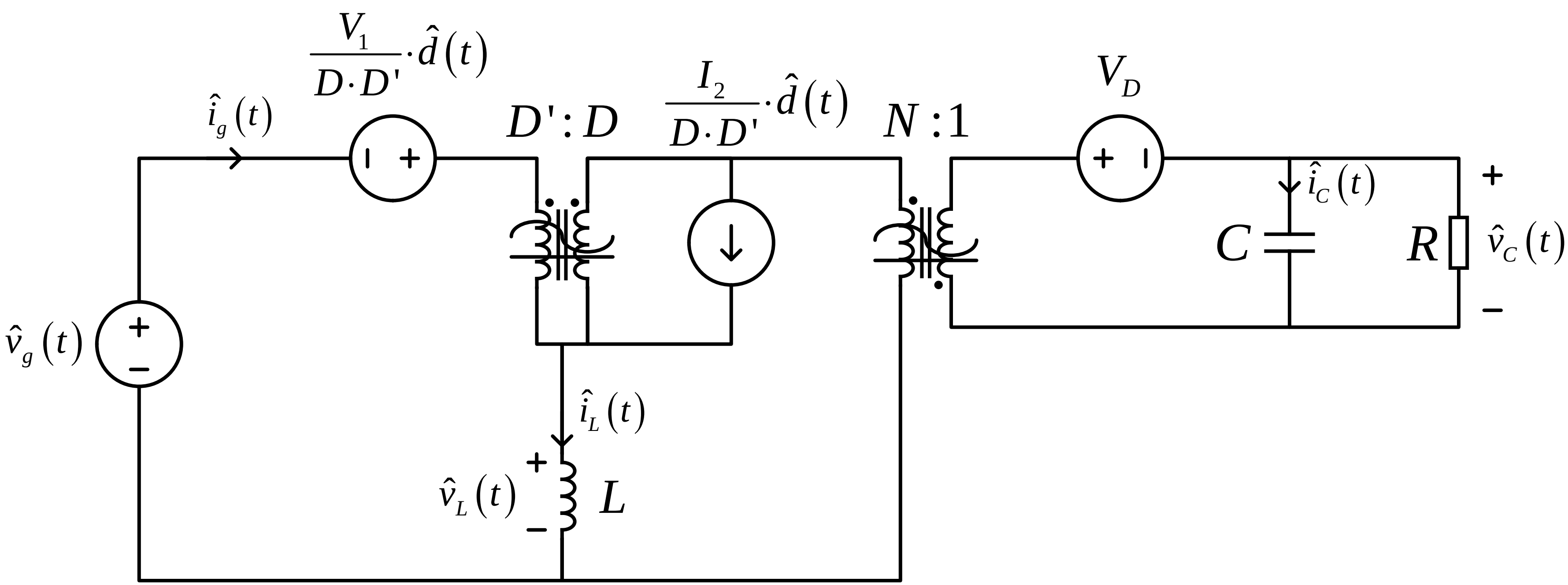


This Looks More Promising

Averaged Switch Small Signal Model: Flyback

!

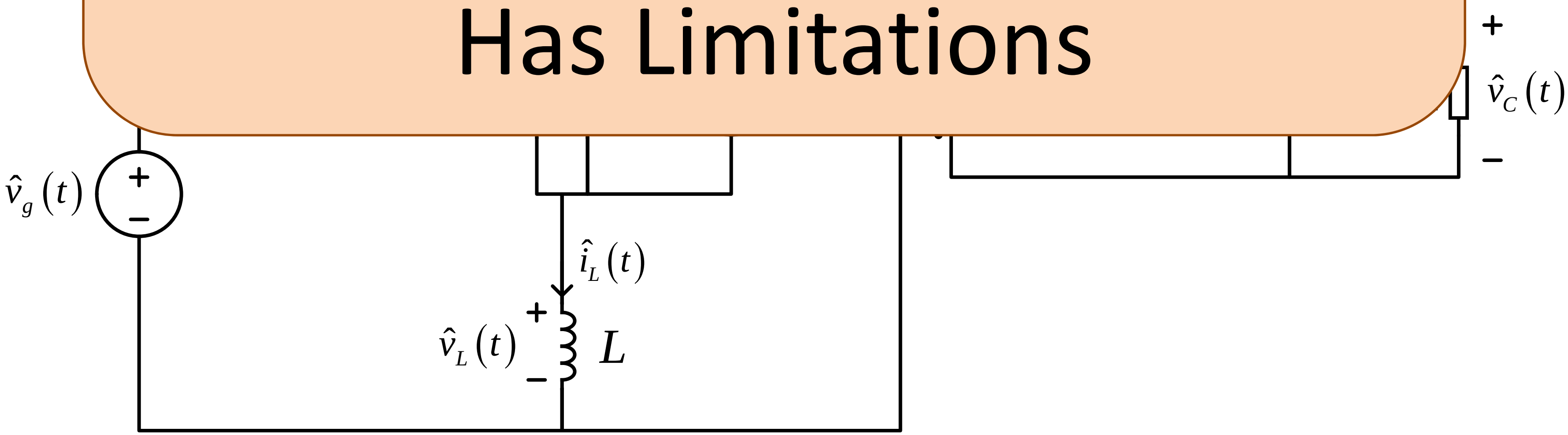
Derive Transfer Functions?
Still Very Tedious



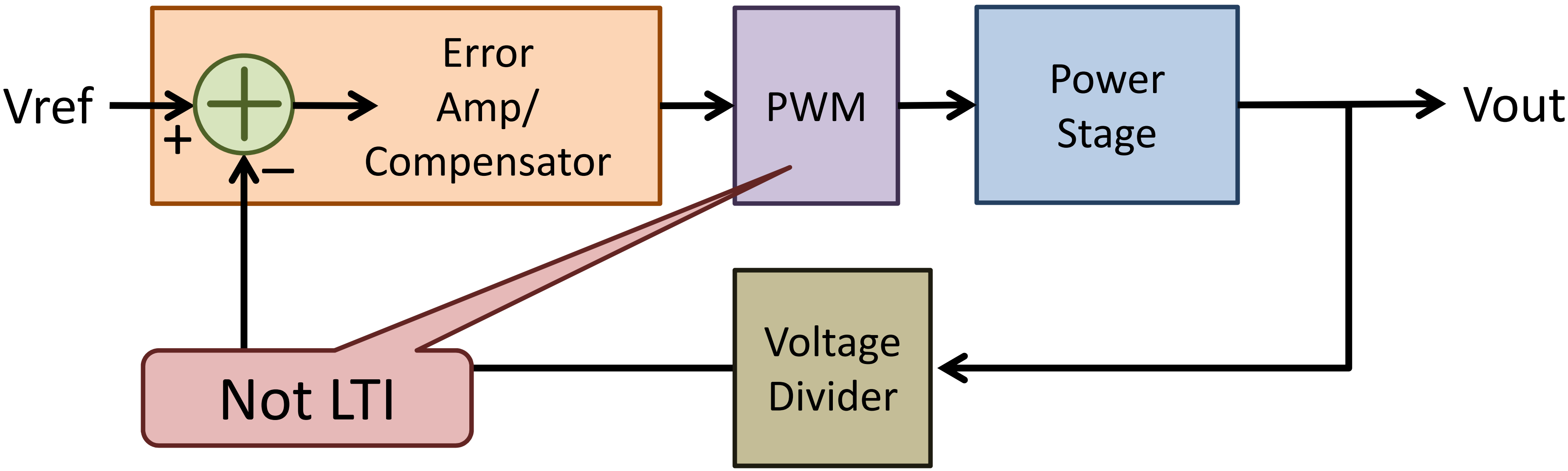
Averaged Switch Small Signal Model: Flyback

!

The Point Of This Example?
Averaged Switch
Small Signal Modeling
Has Limitations



Small Signal Model Of Pulse-Width Modulator



We Also Need A Small Signal Model For The Pulse Width Modulator

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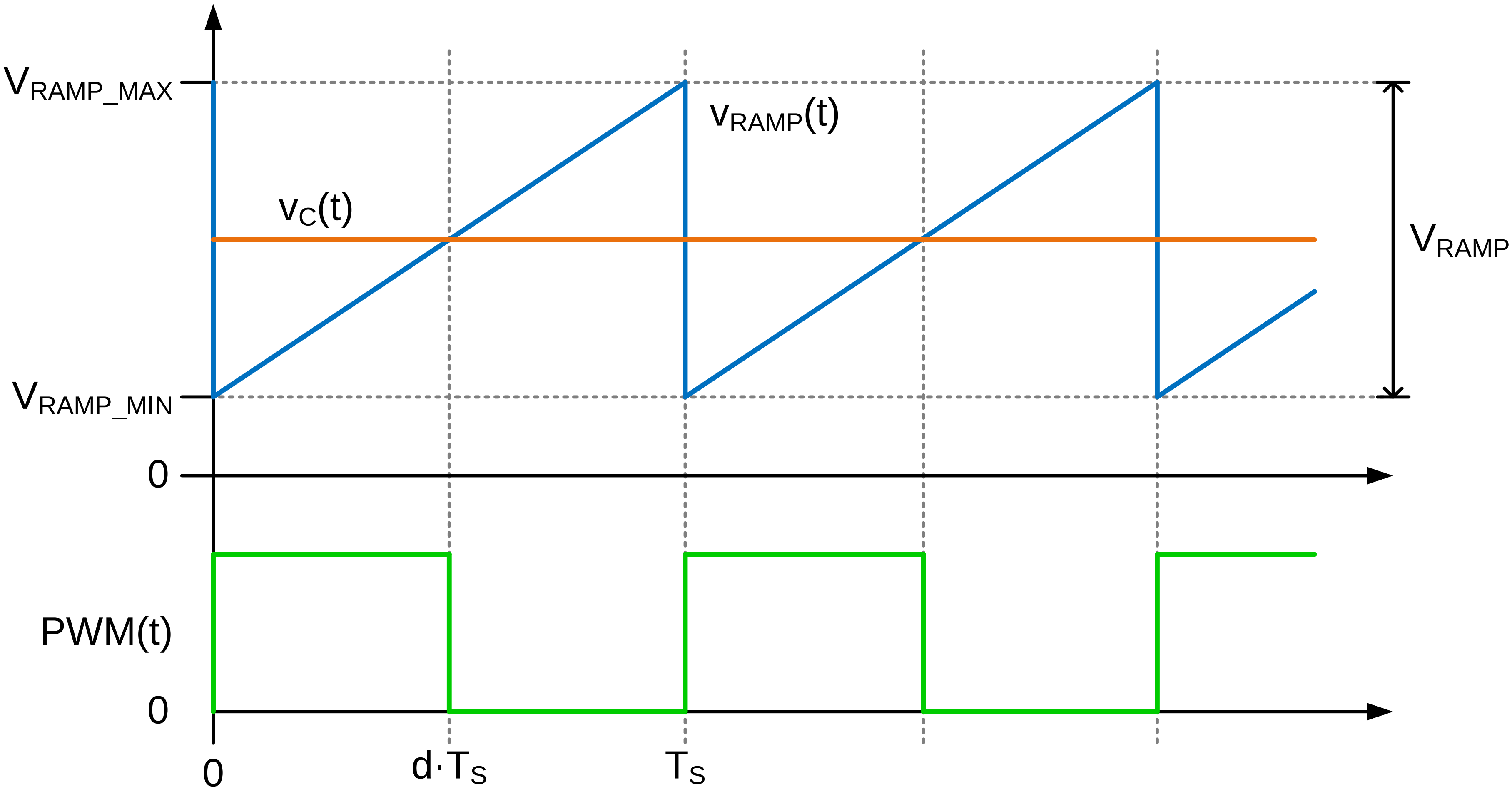
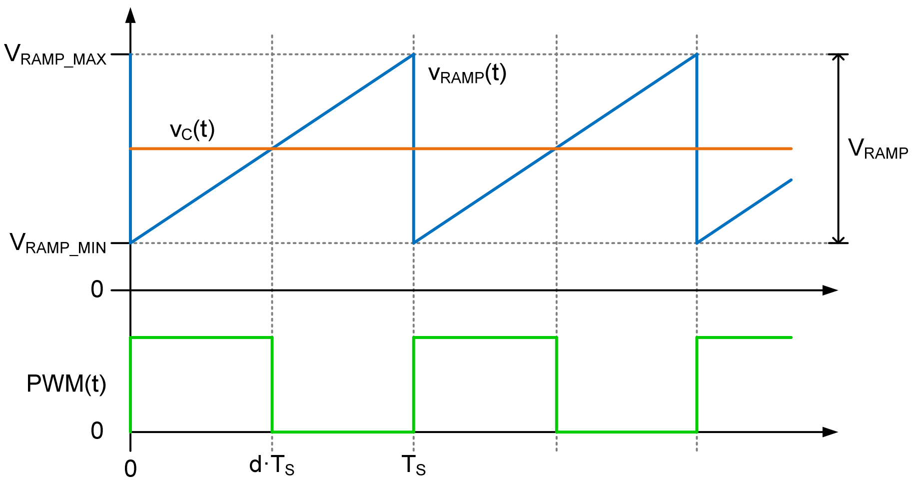
Small Signal Model Of Pulse-Width Modulator



Small Signal Model Of Pulse-Width Modulator



$$d(t) = \frac{v_C(t) - V_{RAMP\_MIN}}{V_{RAMP}}$$

$$= \frac{v_C(t)}{V_{RAMP}} - \frac{V_{RAMP\_MIN}}{V_{RAMP}}$$

$$d(t) = D + \hat{d}(t)$$

$$v_C(t) = V_C + \hat{v}_C(t)$$

$$D + \hat{d}(t) = \frac{V_C + \hat{v}_C(t)}{V_{RAMP}} - \frac{V_{RAMP\_MIN}}{V_{RAMP}}$$

$$= \frac{\hat{v}_C(t)}{V_{RAMP}} + \frac{V_C - V_{RAMP\_MIN}}{V_{RAMP}}$$

$$D = \frac{V_C - V_{RAMP\_MIN}}{V_{RAMP}}$$

$$\hat{d}(t) = \frac{\hat{v}_C(t)}{V_{RAMP}}$$

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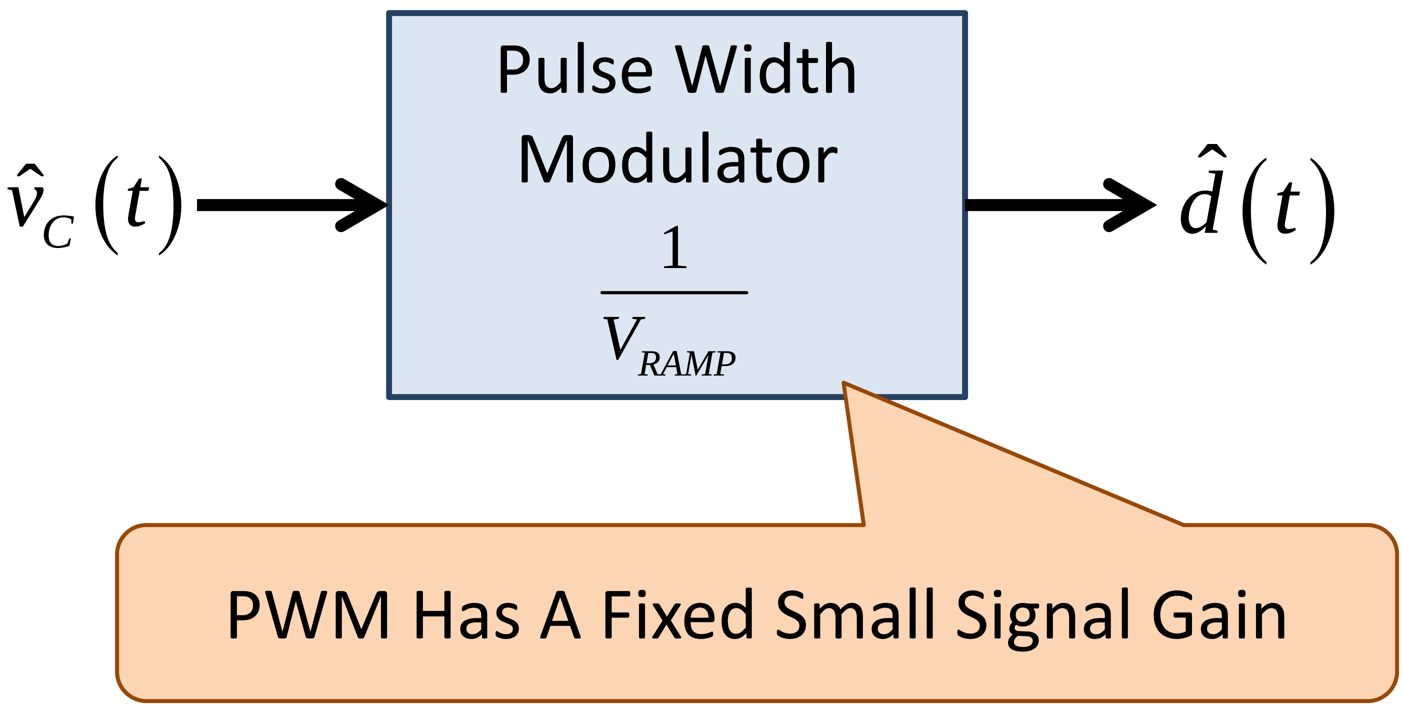
Small Signal Model Of Pulse-Width Modulator

!



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More On Small Signal Modeling

- “Fundamentals of Power Electronics”, 2nd ed., Erickson and Maksimovic, Chapter 7
- V. Vorperian, "Simplified analysis of PWM converters using model of PWM switch. Continuous conduction mode," *Aerospace and Electronic Systems, IEEE Transactions on,* vol. 26, pp. 490-496, 1990.
- V. Vorperian, "Simplified analysis of PWM converters using model of PWM switch. II. Discontinuous conduction mode," *Aerospace and Electronic Systems, IEEE Transactions on,* vol. 26, pp. 497-505, 1990.
- Papers, seminars, books by Ray Ridley and Christophe Basso

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Summary

- We Need Small Signal Models To Use Standard Control Tools To Design The Loop
- Two Averaging Methods Shown To Derive A Small Signal Model Of A Switching Converter:
  - Average Inductor Voltages And Capacitor Currents
  - Average Switch Network Port Voltages And Currents
- Transfer Functions Of Interest Can Be Found From The Small Signal Models

You Can Download The Latest Version Of The Seminar
In Adobe Acrobat (.pdf)
And Microsoft PowerPoint Show Format (.ppsx)
From The Embedded Power Labs website:
http://www.embeddedpowerlabs.com/publications.html

About The Presenter

Bob White has over 30 years experience in power electronics. He has held managerial and individual contributor positions in product development, technology development, applications and systems engineering, and technical marketing. His areas of expertise include power systems for computing and telecommunications systems, digital power, and applications of wide bandgap power semiconductor devices. Bob is currently the president and chief engineer of Embedded Power Labs, a power electronics consulting company.

Bob has been very active in the IEEE Power Electronics Society and the APEC committees, including twice serving as the APEC General Chair.

He is a Fellow of the IEEE, has a BSEE from MIT, a MSEE from Worcester Polytechnic Institute and is currently pursuing a Ph.D. in power electronics at the University of Colorado-Boulder. He is also an Honorably Discharged veteran of service in the United States Air Force.